Physics 2030/2321

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Physics I Laboratory: by Dr. Richard Krantz

Purpose
As a science student it will be assumed upon graduation, or even sooner, (by employers, professors, colleagues, policy makers, etc.) that you have mastered certain skills. In the laboratory these include; 1) problem solving skills, 2) the ability to learn how to use unfamiliar equipment in a timely fashion, 3) the ability to plan and carry out an experimental investigation, 4) the ability to collect, analyze, and interpret data, and 5) to communicate and present your findings in an intelligible way.

1. Problem Solving
Problem solving, in its most compact form, can be summarized by the following four steps: 1) understand the problem, 2) devise a plan, 3) carry out the plan, checking each step along the way, and 4) look back and examine the solution.

Although, upon reflection, these steps seem “obvious” it takes practice to carry out these steps efficiently and expertly for a given problem.

2. Using Unfamiliar Equipment
You will have plenty of practice using, sometimes, unfamiliar equipment in this laboratory. You need not be “afraid” of the equipment. Nor should you be blasé about using the equipment. Pay attention to your instructor about equipment use. You should learn how to use, not abuse, the laboratory equipment provided.

3. Experimental Investigation
An experimental investigation is just another problem to solve. In doing so, you will learn to apply the “Problem Solving” steps listed above.

4. Collect, Analyze, and Interpret Data
Collecting, analyzing, and interpreting data is the motivation for experimental science. The only way we know if our explanations (theories) are justified (correct) is to compare theoretical results to experimental results. If these results are in conflict, assuming the experiment has been done carefully, we must seriously consider that our explanation needs to be amended, or in extreme cases, be thrown out altogether in which case we must develop an altogether different explanation for the phenomenon under examination.

5. Communication
For many of the laboratory experiments you will perform this semester you will be provided with a template for recording data and answering questions pertaining to the experiment. Where appropriate, answers to questions and explanations should be written in Standard English using full sentences. Intelligibility counts, if your instructor or colleagues cannot understand your explanations then your experiment is a failure and your effort has been wasted! Science is a communal activity. To be successful as a scientist you must intelligibly communicate your results.

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3 G. Polya, How to Solve It, Princeton U. Press, 1945)
Measurements, Uncertainties, and Error Propagation

Measurements are a big part of a laboratory. Before we begin the semester, we're going to discuss measurements, uncertainties, and error propagation and include two exercises dealing with measurements.

Measurements are very important in many situations. We use them casually all the time. You say, “What time is it?” I look at my watch and tell you that it’s about 11:50. Did you hear me say “about”? When I looked at my watch I really saw on its display 11:49 and 19 seconds; so the resolution increased as I talked about this new way of looking at the measurement. (More about resolution later.)

There are other measurements that we take every day. Consider a ruler, just a straight edge: there’s a metric scale, and it counts from 1 to 45 centimeters; the other side does inches; there are all sorts of little marks representing millimeter, ¼ inch, 1/8 inch, and 1/16 inch. This one little ruler has much to say.

In the laboratory, we don't take measurements for granted. We focus on them. We want to understand each measurement.

How well do we really know a measurement?

Six Steps to Taking a Measurement

1. We first need an “object” to measure
2. Standard of Measure
3. Unit of Measure
4. Measurement Procedure
5. Uncertainty
6. Confidence Factor

1. We first need an “object” to measure

What is involved in taking a measurement? First, we must have an object or something that needs to be measured. It could be anything; it could be as simple as a metal rod. But an "object" can sometimes be a little elusive. What if we want to measure light? We can't grab light and measure it in our hand.
You may have noticed in discussing the object “rod” and the “object” “light” we did not specify what we would measure: length, mass, speed, bending ability, stretching ability, conductivity, energy, frequency, and the list continues.

2. Standard of Measure

This brings us to the second step: a standard of measure. Standards are not easy to create nor obtain. They are very expensive to maintain. We can buy a cheap standard from the drugstore down the street. The ruler has markings that denote inches on one side and centimeters/millimeters on the other. If it is a good standard, it’s been compared to a standard accepted around the world. I have compared a ruler from a drugstore to a standard purchased from a scientific company. The ruler from the drugstore had many discrepancies when compared to the scientific company’s standard. Some of the markings were elongated; some were shorter. I could barely believe my eyes.

In the United States, standards are held by NIST (National Institutes of Standards and Technology). NIST is based in Boulder, Colorado. At NIST, they actually 'hold' the standard for the meter. The meter is not held on a rod which is kept at a certain temperature. Measurements for the meter are kept at NIST as the distance traveled by light in a certain time frame.

“Today, the meter (m) is defined in terms of constant of nature: the length of the path traveled by the light in vacuum during a time interval of 1/299,792,458 of a second.”

(Nist.gov) http://www.nist.gov/pml/wmd/metric/length.cfm

Any person or company can arrange with NIST to compare their “rod” with the standard of the meter. Then they can take their rod back to their manufacturing plant and process measuring devices based on that standard.

NIST also holds the standard for the second. NIST has a cesium atom that vibrates. 192,631,770 Hertz: this is how many vibrations a Cesium atom does in 1 second. The uncertainty in the atomic clock is on the order of 1.7E-15, or accuracy to about 1 second in 20 million years.

(Nist.gov) http://tf.nist.gov/cesium/atomichistory.htm

NIST takes that time measurement and sends it to their timeserver in Fort Collins, where the timeserver is located for the entire US, and a radio signal is sent out. Atomic clocks receive that signal and reset themselves according to that signal, so that they are accurate, or agreeing with the time standard. I remember as a young man calling the timeserver with watch in hand. I did this frequently. The timeserver would answer and say “Tick, tick, tick,” until there was a long beep and then the time would be announced (The older I get, the fuzzier this memory becomes). So, call NIST yourself and listen today: (303) 499-7111.

“Beep, beep, beep, ... tick, tick, tick, ...At the tone, thirty-eight minutes coordinated universal time, beeeeeeeep.”
**NIST Telephone Time-of-Day Service**

The audio portions of the WWV and WWVH broadcasts can also be heard by telephone. The time announcements are normally delayed by less than 30 ms when using land lines from within the continental United States, and the stability (delay variation) is generally < 1 ms. When mobile phones or voice over IP networks are used, the delays can be as large as 150 ms. In the very rare instances when the telephone connection is made by satellite, the time is delayed by more than 250 ms.

To hear these broadcasts, dial (303) 499-7111 for WWV (Colorado), and (808) 335-4363 for WWVH (Hawaii). Callers are disconnected after 2 minutes. These are not toll-free numbers; callers outside the local calling area are charged for the call at regular long-distance rates.

The telephone time-of-day service is used to synchronize clocks and watches and for the calibration of stopwatches and timers. It receives about 2,000 calls per day.


Notice how NIST doesn’t just tell you the phone number. What details are mentioned first? Notice also how they only tell you the current minute. What is the benefit to not specifying the hour?

Standards are used all the time (no pun intended) and agreement with standards is important. If I want my clock to be accurate, I must compare it to the standard clock held by NIST. Then I can say the time is accurate: 11:57 and 15 seconds ± less than one hundredth of a second because I know both **precision** and **accuracy**.

Consider these different levels of precision for measuring time:

- About 12 o’clock (Precision at *hour* level)
- 11:57 (Precision at *minute* level)
- 11:58:02 (Precision at *second* level)
- 11:58:02.87 (Precision at *hundredth of a second* level)

(More on precision later.)
3. Unit of Measure

Next, we need a **unit of measure**: millimeters, centimeters, inches, Newtons, pounds. NIST.org is a great place to learn about units.

The following graphic is from NIST:
http://www.nist.gov/pml/wmd/metric/si-units.cfm

Great catastrophes have occurred because two people were measuring with different units. Try to find about “NASA’s metric confusion caused Mars Orbiter loss.”

4. Measurement Procedure

How we go about taking the measurement can greatly impact the final answer for the measurement. We will see more about this later.
5. Uncertainty

Then there is the **uncertainty** of the measurement. How well did we actually know the measurement? We can only give a best estimate for our measurement of the object based on the standard that has a unit. Using our procedure for measurement, we’ll find out the uncertainty of the measurement. There is an uncertainty in every measurement that we take; we don’t want to take that for granted. Measurements have a best estimate according to the scale, and then we evaluate the uncertainty. In a laboratory we are interested both in the measurement and the uncertainty.

6. Confidence Factor

**Confidence factor** is the uncertainty in the uncertainty. How much confidence do we have in our uncertainty? We will not deal with this topic other than just to mention it.

---

**How do we find the value for the uncertainty?**

During the semester, you will be taking measurements. With each measurement you need to stop and consider the uncertainties. The uncertainties are critical for properly reporting your measurements. The lab manual will not ask you to stop and consider your measurement. We are asking you to stop and consider each measurement.

After establishing the standard and its unit, and your measurement procedure, this is where you stop and ask the questions pertaining to uncertainties. Essentially, there are three questions that you can ask to find the uncertainty value. Before we talk about these three questions, let’s remember that an uncertainty most often will match the unit used for the measurement and it is reported with a ± symbol.

For example: Length = 2.41[m] ± 0.02[m]

**Three questions to ask with each measurement in order to find the measurement uncertainty:**

1. **Resolution**
2. **Interface**
3. **Repeatability**

One of these questions will tell us the value for the uncertainty.

**1. Resolution:** What does the measuring device tell you about its resolution? The readability is the resolution. This answers the question of precision and it can tell you the uncertainty. If your stopwatch can be read to 0.01[s], that is the readability or the resolution. And the uncertainty is ±0.01[s].
The uncertainty or readability of an analog measurement can be taken as ±1 the smallest division. With a ruler, the measurement is a two-point measurement, and with our eyeballs we can resolve to ½ of the smallest measurement (for each end of the object). When the object sits in between two of the markings, we can determine it is close enough to halfway. Some disciplines take the smallest division and divide it up into ten parts. They then call that the "best guess" for the next decimal place. For physics, the analog measurement is readable to ½ the smallest marking, and we talk about the "best guess" within our uncertainties. And since length is a two-point measurement, the uncertainty is doubled, making uncertainty ±1 unit.

2. Interface (Interaction) with the measurement: how did I make the measurement? What was my procedure? Does this procedure intrinsically make the uncertainty bigger?

Two examples for how interface may or may not change the uncertainty value:

a) Measuring with a Digital Multimeter (DMM): this involves hooking probes up to a resistor and taking a measurement, in which the interaction is not important. You can take the measurement over and over again and you get the same measurement every time. Your lab partner can do the measurement; the same answer is revealed. Connecting the probes and connecting the resistor doesn’t seem to change the measurement. So, in this case uncertainty reverts back to Resolution or Readability of device.

(Most reliable DMM’s account for the probe wires internally; but if you switch out the original wires, then you have to carefully account for resistance in the wires.)

b) Measuring the length of a rod: this involves procedure. How I interface the meter stick with the rod impacts the uncertainty. Some factors that impact the uncertainty:

- not beginning with the end of the measuring device, but rather at a non-zero point
- is my meter stick touching the rod?
- is the scale touching the rod? (If the meter stick or rod have thickness, then the scale markings might not touch the rod.)

- **Parallax**: this is when the scale markings are not touching the object (and more specifically, not touching each or one end of the object). When you look at the object and compare its end to the scale markings, as you move your head, the measurement seems to change. This is parallax. One way to test for parallax is to measure with one eye shut. Then, without moving your head, switch eyes. If the measurement changes, parallax exists. This is because you cannot be certain your line of sight is perpendicular to the plane holding the scale markings.

If something in my procedure causes greater uncertainty than the readability allowed, I **must** use the larger uncertainty.

If my two eyes read different values by as much as 2[mm], then the resolution of 1[mm] is overruled by the procedure (interface) and the uncertainty is ±2[mm].

A stopwatch can measure to 0.01[s]. But if my reaction time is 0.2[s], then I have to use the interface question to answer the uncertainty question. And since time of the
Measurements, Uncertainties, and Error Propagation

event requires two presses of the stopwatch, my uncertainty becomes ±0.4[s]. That is 40 times larger than the resolution questions yields.

3. **Repeatability (Standard Deviation)** Requires many samples, preferred.

In this sample experiment we will measure how long it takes a cart to travel a set distance down a fixed incline. We will use a digital stopwatch. It reads to 1/100th of a second.

**Resolution** speaks to precision and uncertainty through the readability. The stopwatch measures to 1/100th of a second.

Resolution: 1/100th of a second. ± 0.01 [s]

**Interface:** Suppose the user’s reaction time is 0.14 [s] → 0.2 [s]*2 → ±0.4[s]

**Repeatability:** Repeat the experiment 30 times. This allows you to find the standard deviation. The standard deviation is a statistical method and is most often trusted more than resolution and interface. If you can repeat a measurement and get slightly different results each time, repetition and subsequent standard deviation is the preferred answer to the uncertainty question.

The Standard Deviation (σ) answers this question:
How far from the mean value (±) will 68.2% of all the measurements lie?

\[
\text{Standard Deviation: } \sigma = \sqrt{\frac{\sum(L_i - \bar{L})^2}{N - 1}}
\]

The **standard deviation** is the square root of the ratio whose numerator is the sum of differences squared and whose denominator is the number of data points minus one. The differences being squared (and subsequently summed) are found by taking the difference between the average of all the data points and each individual data point. In the example above \( \bar{L} \) stands for a measurement in length.

The symbol for standard deviation is lower case sigma: \( \sigma \).
The symbol for summation is upper case Sigma: \( \Sigma \).
\( \Sigma \) allows for “shorthand” notation.

\[
\sum_{i=1}^{n} (L_i - \bar{L})^2 = (L_1 - L_{avg})^2 + (L_2 - L_{avg})^2 + (L_3 - L_{avg})^2 + \cdots + (L_n - L_{avg})^2
\]

\( L_i \) is the i-th data point where \( i = 1 \) to \( n \) (and \( n \) is the number of data points).
\( \bar{L} \) is the average value for all the \( L \) data.
\( N \) is the number of data points.
68.2% of all the data will fall within ±1σ of the mean value. If the mean value is 2.4[s], and the formula yields σ = ±0.3[s], then this means that 68.2% of all the data points will fall between 2.1[s] and 2.7[s]. This is written as 2.4±0.3[s].

±1σ gives us 68.2% of all the data.
±2σ gives us 95.4%
±3σ gives us 99.7%
±4σ gives us 99.99%
±5σ gives us 99.9999%

On the following page is an example showing a measurement with uncertainty. The measurement was repeated 100 times. Much of the data (about 68%) falls within a certain range as measured from the mean value. This range is ±σ. The measurement for this example is

Length = 2460[cm] ± 30[cm]
Probability Distribution

This probability distribution curve shows a bell shape. This curve is also called a Normal Distribution or Gaussian. 68.2% of the data will be within plus or minus the standard deviation (±σ) from the mean value.

If we go out to ±2σ, then 95.4% of the data is within that range from the mean.

±3σ gives us 99.7% of the data points.

Length = 2460 ± 30 [cm]
σ = 30 [cm]

100 data points were taken. 68.2% of all the data is within ± 30[cm] from the mean of 2460[cm].
Measurements, Uncertainties, and Error Propagation

Analog and Digital Measurements:

For analog scales, use ±0.5 the smallest marking [unit] to express the uncertainty factor. But, because analog measurement is usually a 2-point measurement, you must use ±0.5 for each point, therefore, the uncertainty becomes ±1.0 the smallest unit.

Digital measurements will vary from multimeters to stopwatches to vernier calipers to micrometers. Normally, the resolution (readability) of the digital scale is the uncertainty. In the lab, be careful for computer measurements and calculations. DataStudio™ and MSExcel will often report 5 or more significant figures; it is the student’s job to know how many digits are truly significant. (MSExcel stores on the computer somewhere around 13 digits, many of which are hidden from the eyes of the user.) Some digital devices count by one’s, two’s, or 5’s. Be careful to not assume the smallest digit is counting by one’s. Good manufacturers know the precision of the instruments they sell; when in doubt, contact them.

Human error

*Human error* is a phrase not to be used in the laboratory! It is obvious that humans are doing the experimentation. Use the phrase "reaction time" instead. Human error will make your instructor to assume you made a mistake in the procedure and mark your assignment down. On the other hand, if you did make a mistake, repeat the experiment and correct your mistake. Be specific about how the human interacted with the equipment to introduce the uncertainty and speak to the specific example.

Special Note About Recording Numbers and Units

Two personal *style points* I tend to use:

When typing a unit of measure use [ ] to enclose the unit. This aids in keeping units separate from variable names and numbers. It also differentiates calculation grouping ( ) from units. 0.034[s].

Notice how the square brackets [ ] are easily typed without modifier keys.

When writing numbers less than unity, always use a leading zero. 0.467[m/s/s] - Notice how your eye may have an easier time seeing the decimal when compared to .467[m/s/s].
**Precision vs Accuracy**

**Precision** and **accuracy** are not synonymous!

For *precision*, think of the word "pricey." A ruler costs $0.30 at the drugstore and is not very precise. A micrometer is more precise. It has more decimal places and can measure smaller objects. **Precision** speaks to how many digits (place values) are represented in the measurement. The greater the precision, the closer repetitive measurements will be to one another.

**Accuracy** is comparing your measurement to the standard (the accepted value). It is asking, "How close to the mark did we come?" In the case of accuracy, repetitive data can be vastly different from one another, yet their average is still describing the accepted value.

On the following page is a common way to display the difference between precision and accuracy.
**Significant Figures**

When you think of significant figures, think of the word precision. Think of how many decimal places (or digits) you can actually measure. One should not report a measurement using more digits than what was actually measured (or accounted for through error propagation).

It is important to note when discerning significant figures that we do not count the placeholder called zero.

Consider the following numbers:

- **0.00123** has 3 significant figures. The zeros are not significant. They simply place the measurement at the right place value.
- **1230** has 3 significant figures. Again the zero is placeholder to show the actual measurement started at the 10’s place.
- **1230.** has 4 significant figures. Adding a decimal makes the zero significant.

You can also use scientific notation. In scientific notation, all digits revealed are significant.

Let’s compare these two numbers:

- **1.230E+3** has 4 significant figures. The zero shows that the measurement was made to the one’s place.
- **1.23E+3** has 3 significant figures.

Think of 1.230E+3 as a measurement taken in meters with a meter stick. The length of this meter stick was 1 meter long. It did not have any smaller divisions; it was simply a stick that was 1 meter long. The zero in this number was significant because it is in the one’s place, and we were measuring with a 1-meter stick.

The number 1.23E+3, on the other hand, was measured with a 10-meter stick. This stick was 10 meters long and had no smaller divisions within it. With that stick, we could not measure single meters, but tens of meters.

- **1.0456** has 5 significant figures.
- **1.04560** has 6 significant figures. The zero added at the end is significant because it is not a placeholder; this last digit (which happens to be zero) does not decide where the measurement resides (in place value).

Significant figures are important in measurements because they tell you the **precision** of your measurement.
Example:
In measuring the distance between Denver and the West Coast, you will probably not use a micrometer. You will use a device that measures in kilometers. Your precision will not extend to meters nor centimeters.

Someone claims they measured the distance from Denver to Los Angeles to be 1609.344 [km]. This would be a very costly measurement, because you would need a very expensive instrument to measure in meters precisely all the way from Denver to the West Coast. (This measurement 1609.344 [km] is 1609 km and 344[m].) It is not likely that such a measurement was reported correctly, because the measurement is too precise! A realistic measurement would be simply 1609[km] ± 1[km].

Scientific notation is a good way to talk about measurements because the number of significant figures is specified within the writing of the number.

4.56E-4 = 0.000456
The zeros are not significant because they are only placeholders.

A common question is, "How many significant figures do I employ in my final result?" or "How many significant figures do I employ in this particular measurement?" In a physics laboratory, the answer to significant figures comes from either uncertainty or error propagation. We will discuss both in the exercises that follow soon.
So, How Different Are They? Or, What Is Their Discrepancy?

When we have two different representations of a particular value, we need some way to compare the two values. We can use either Percent Difference or Percent Discrepancy.

For example, we have measured the mass of an object with a digital balance and with an analytical balance. Neither measurement is necessarily more accurate than the other one. In this case, we use Percent Difference.

\[
\text{Percent Difference} = \left( \frac{|\text{Measurement}_1 - \text{Measurement}_2|}{\text{Average of } M1 \text{ and } M2} \right) \times 100\%
\]

In another example, we have measured \( g \), the acceleration due to gravity. But \( g \) has a value that is well accepted by scientists around the world. Let’s say NIST has even measured \( g \) at your experiment location. (There are companies that will bring a device to your laboratory and measure the value of \( g \) to \( n \) decimal places, where \( n \) is defined by your pocketbook.)

You measure \( g \) to be 9.67[ms\(^{-2}\)];
but the accepted \( g \) is 9.80[ms\(^{-2}\)]. (We also call this the expected \( g \).)

In this case we use Percent Discrepancy.

\[
\text{Percent Discrepancy} = \left( \frac{|\text{Measurement} - \text{Expected}|}{\text{Expected}} \right) \times 100\%
\]

\[
\text{Percent Discrepancy in } g \text{'s} = \left( \frac{|9.67 - 9.80|}{9.80} \right) \times 100\% = 1.3\% \text{ Discrepancy}
\]

Notice that both Percent Difference (no expected value) and Percent Discrepancy (expected value) will be positive (not negative) because we use the absolute value of the difference.

Finally, note the term Relative Difference is the same as Percent Discrepancy.
**Experiment 1: The Entire Class Will Measure a Rod and Report Its Average Measurement with Uncertainty**

Each student will measure a rod, write down the measurement, record the measurement into a worksheet. The instructor will provide rod, measuring standard, and worksheet.

Analysis: For this experiment, the analysis will include the average, number of data points, variance, standard deviation, and standard deviation of the mean.

Equation for Average Length:

\[
L_{\text{avg}} = \frac{\sum_{i=1}^{n} L_i}{n}
\]

Equation for Variance:

\[
V = \frac{\sum (L_1 - \bar{L})^2}{N - 1}
\]

Equation for Standard Deviation of the Mean:

\[
\sigma = \pm \sqrt{V}
\]

**Experiment 2: Individually Measuring the Area and Perimeter of a Lab Table and Determining Uncertainties and Error Propagation**

Students will calculate the area and perimeter of a table by finding the width and length of the table.

Important reminders: You must use a 1-meter stick. You may not switch meter sticks. You must measure area and perimeter of the table using the outside edge of the table. Worksheets for this experiment can be found at the end of this lab entry.
How do uncertainties propagate through the equation?

The symbol for uncertainty is \( \delta \) (lower case delta). We use \( \delta \) because upper case Delta \( \Delta \) means difference. In the case of uncertainties, we are talking about small differences, so we use a lower case delta.

Remember:
Measurement Uncertainty is found from answering one of three questions:
Measurement Uncertainty \( \delta_m \)

\[ ? \rightarrow (1) \text{ Resolution} \quad (2) \text{ Interface} \quad (3) \text{ Repeatability} (\sigma) \]

**Measurement uncertainty** for perimeter can be found by adding the measurement uncertainties for both width and length. When we measure width, the uncertainty is derived from both sides of the meter stick. We simply add the uncertainties; one uncertainty from each side of the stick. The same goes to reason for the perimeter. We are simply adding measurements; so, we can simply add the uncertainties.

Measurement Uncertainty for Perimeter of Table: \( 2\delta_l + 2\delta_w = \delta_p \)

Of course, the uncertainties must be doubled because there are 2 widths and 2 lengths.

**Error Propagation for Sums**: Sum the uncertainties whenever you have a sum of measurements. The sum will become the uncertainty in your final answer.

**Relative uncertainty** relates the uncertainty to the measurement. It is the ratio of uncertainty in the measurement to the measurement itself which is then multiplied by 100%.

\[
\text{Relative Uncertainty as a Percentage} = \frac{\delta_m}{m} \times 100\%
\]

To find the relative uncertainty in the width of the table, take the uncertainty in the width and divide by the width, which gives us a dimensionless number, relating the uncertainty to the measurement.

\[
\text{Relative Uncertainty of Width as a Percentage} = \frac{\delta_w}{w} \times 100\%
\]

Follow the same procedure for finding the relative uncertainty of the table length.

\[
\text{Relative Uncertainty of Length as a Percentage} = \frac{\delta_l}{L} \times 100\%
\]
**Relative Uncertainty** for perimeter can also be found by adding the relative uncertainties for each measurement.

Relative Uncertainty for Perimeter of Table: \[ \frac{\delta_L}{L} + 2 \frac{\delta_w}{W} = \frac{\delta_P}{P} \]

**What about the Uncertainty for Area?**

Error Propagation: As we manipulate measurement variables with various equations what then happens to the uncertainties? Do they grow? Do they stay the same? Do the uncertainties shrink? Wouldn’t that be nice? Take a measurement, square the measurement, and watch the uncertainties get smaller. I think not...

We measure the side of a square to be 5.6[cm]±0.3[cm] and then want to find the area of the square.

\[ A = 5.6 \times 5.6 \text{[cm*cm]} = 31.36\text{[cm}^2\]. \]

What happens to the uncertainty in our measurement: ±0.3[cm]? How does the uncertainty propagate through the area calculation?

We multiplied the measurements to find area; so let’s multiply the uncertainties to see what happens:

\[ 0.3 \times 0.3\text{[cm*cm]} = 0.09\text{[cm}^2\]. \] Ah, oh! The uncertainty in area looks like a smaller number. I realize they are different units, so let’s compare relative uncertainties:

\[ \frac{0.3}{5.6} \rightarrow 5.4\% \]

\[ \frac{0.09}{31.36} \rightarrow 0.3\%; \] this can’t be! Uncertainties certainly cannot get smaller as we find the area of the square.

So, instead of multiplying the uncertainties, we will add the uncertainties just we did for the perimeter. But wait! What if we were multiplying Mass and Velocity to find Momentum? We can’t add \textit{kg} and \textit{m/s} to find the uncertainty in \textit{p} (momentum). But adding Relative Uncertainties will work. Remember, relative uncertainties are dimensionless; this allows us to add relative uncertainties for every time we take a product with our measurements.

\[ \frac{\delta_p}{p} = \frac{\delta_{\text{mass}}}{m} + \frac{\delta_{\text{vel}}}{v} \]

This is all well and good; but be careful for when measurements are really calculations. If velocity was a calculation based upon time and displacement, then

\[ \frac{\delta_p}{p} = \frac{\delta_{\text{mass}}}{m} + \frac{\delta_{\text{time}}}{t} + \frac{\delta_{\text{dis}}}{d} \]

**Error Propagation for Products**: Sum the Relative Uncertainties whenever you have a Product of Measurements. The sum will become the relative uncertainty in the product.
Measurements, Uncertainties, and Error Propagation

For the Area calculation (a product) we can find the uncertainty in area as a Relative Uncertainty:

$$\text{Relative Uncertainty in Area} = \frac{\delta_A}{A} = \frac{\delta_L}{L} + \frac{\delta_W}{W}$$

Adding relative uncertainties is the procedure followed whenever measurement variables are multiplied. Notice the power this gives us when we have variables with different units. The relative uncertainties are dimensionless. And, in the case when a variable is squared, we simply double the relative uncertainty.

For example, we want to find \( g \), the acceleration due to gravity. To find \( g \), we measure displacement, height, length, and time\(^2\). So the total relative uncertainty in \( g \) is simply the sum of all the relative uncertainties for all the variables that were used. Take special care to double the relative time uncertainty since time is squared in the analysis.

$$\text{Relative Uncertainty in } g = \frac{\delta_g}{g} = \frac{\delta_L}{L} + \frac{\delta_H}{H} + \frac{\delta_D}{D} + 2 \frac{\delta_t}{t}$$

To find \( \delta_g \), the measurement uncertainty for \( g \), we multiply our relative uncertainty in \( g \) by \( g \) itself. By \( g \), we mean the \( g \) that we found during the experiment through calculation and statistics.

$$g \times \text{Relative Uncertainty in } g = \delta_g$$

How Many Significant Figures Do I Use?

Notice what follows might be different from what you were taught when you first learned about significant figures. You remember the rules, don’t you? For a sum we use the largest number of significant figures. For a product we use the least number of significant figures. But that is for mathematics. What about when the numbers represent measurements?

The question of how many significant figures are needed for a measurement or a final result can be found in the uncertainties. The uncertainties tell you to what decimal place you may report.

Suppose the table length is 96.253m and the uncertainty in the length is \( \delta_L = 0.02m \).

You should see a discrepancy. For our labs, uncertainties will typically be one significant figure, and that one significant figure answers two questions:

To what decimal place am I allowed to measure?
At that decimal place, how do I count?

The correct way to report this measurement is 96.25±0.02m. The 0.02m tells me to what level of precision I measured and how many significant figures I need to write.
Measurements, Uncertainties, and Error Propagation

Example
Finding area: Suppose we determine the area of the table to be 2563.272 m$^2$. We determine the relative uncertainty of the area to be $\frac{\delta A}{A} \times 100\% = 4\%$. We multiply the uncertainty of the area by the area itself and then determine the measurement uncertainty in area is $\pm 100\text{m}^2$. ($4\%$ of 2563.272 $\approx$ 100. Remember, uncertainties this semester are typically one significant figure. So $0.04 \times 2563.272 = 102.531 \rightarrow 100$.) Therefore we report the area as:

$$\text{Area} = 2600[\text{m}^2] \pm 100[\text{m}^2].$$

Notice we did not report the 63.272[\text{m}^2]. Our error propagation calculations told us we could only report to the nearest 100[\text{m}^2]. And notice the 2560 was rounded up to 2600.

In Review
Measurements are to be carefully considered this semester. Stop when taking each new type of measurement and answer these questions:

What “object” am I measuring?
What is my standard?
What are my units?
What is my procedure for measurement?
What is the uncertainty in this measurement?
Ask:

1. Resolution
2. Interface
3. Repeatability: Standard Deviation

How many significant figures am I going to report for my measurement? Look to the uncertainties.

How many significant figures am I going to report for my final answer? Sum the Relative Uncertainties for your final analysis.

Please don’t think that measurements are cheap nor that measurements should be taken casually. Measurements are costly with time, money, and thought. It is a process to get a measurement. Don’t forget that errors propagate through calculations. They propagate through multiplication by simply summing the relative uncertainties.
# Measurements

Lab Station Number: 

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<th>First Name</th>
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</table>

Names Sorted Alphabetically (by last names)

Date Data Was Taken: 

Instructor's Name: 

Lab Time: 

*Examples: T8, W4, R10*

**Pages are stapled in upper left corner and are in this order (5pts):**
- Measurement of Stick (Rod)
- Measurement of Perimeter and Area of a table
LENGTH OF A STICK – Statistical uncertainties
Each student will measure a stick using the same standard.

<table>
<thead>
<tr>
<th>Student</th>
<th>( L_i ) [cm.]</th>
<th>((L_i - L_{\text{av}})) [cm.]</th>
<th>((L_i - L_{\text{av}})^2) [cm.]^2</th>
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</thead>
<tbody>
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\[ L_{\text{av}} \text{[cm]} = \frac{\sum L_i}{N} \quad \text{BEST ESTIMATE!} \]
\[ \sigma_{\text{cm}} = \sqrt{\frac{\sum (L_i - \bar{L})^2}{N(N-1)}} \quad \text{STANDARD UNCERTAINTY} \]

\[ \sigma_{\text{m}} = \sqrt{\frac{\sum (L_i - \bar{L})^2}{N(N-1)}} \quad \text{(Standard Deviation of the Mean)} \]

REPORTING: \( L_{\text{av}} \pm \sigma_{\text{m}} \) \quad YOUR RESULT ______________________

MEANING (INTERPRETATION): Your result has about a 68% chance of being between
\((L_{\text{av}} - \sigma_{\text{m}})\) and \((L_{\text{av}} + \sigma_{\text{m}})\). Or, the length is between \((L_{\text{av}} - \sigma_{\text{m}})\) and \((L_{\text{av}} + \sigma_{\text{m}})\) with about 68% confidence.
Measure Perimeter and Area of Table:

Length Measurement Tool is: __________________________

Resolution of said device: _____________________________

Resolution might be used as the uncertainty of measurement.

Procedure for measuring width (short distance)

Procedure for measuring length (long distance)

<table>
<thead>
<tr>
<th>Width [ ]</th>
<th>Uncert. [ ]</th>
<th>Length [ ]</th>
<th>Uncert. [ ]</th>
<th>Perimeter [ ]</th>
<th>Area [ ]</th>
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Relative Uncertainty:

Relative Uncertainty in:

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Measurement Uncertainty in:

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Write these out fully:

Length ____________________________

Width ____________________________

**How do uncertainties propagate through calculations:**

Perimeter: the SUM of measurements; but what about the uncertainties?

Area: which is the product of measurements; but what about the uncertainties?

<table>
<thead>
<tr>
<th>You have L &amp; W from earlier</th>
<th>These are new calculations</th>
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<tbody>
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<td>Average</td>
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<td>Relative Uncertainty</td>
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Write these out fully:

Perimeter ____________________________

Area ____________________________
What about the whole class:

**Standard Deviation:**

<table>
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<th>Table 1</th>
<th>Perimeter</th>
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<td>Stdev</td>
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</table>

**Three questions to find uncertainty for each measurement:**

1) Resolution (Readability: digital vs analog)
2) Interface (Interaction: parallax, reaction time, perception range, etc.)
3) Repeatability: Standard Deviation (Many Samples. Preferred.)
Data & Graphs

Purpose

We will use data from an airtrack glider to find the relationship between time and distance fallen. We will employ graphical techniques to investigate this relationship. The exercise in its entirety is lengthy. Follow the write-up carefully to gain insight into how you should record information during the semester.

Theory

Most of the experiments in this semester of physics are based upon theoretical equations. Often times those equations were borne out of someone taking much data of an unexplained phenomenon. Relationships between parameters of the experiment were found by seeing how a change in one variable caused a change in the others. Eventually, an equation is developed that relates the variables in question. This equation can then be tested through further experimentation of different types.

The unexplained phenomena for this lab is as follows:

Displacement and time seem to be related to one another for a glider sliding down a frictionless incline. How are displacement and time related?

We can imagine that a short fall takes less time than a longer fall. If we drop a ball it moves faster and faster and faster. But to measure the time for a ball to drop one, one and one-half, and two feet is a serious challenge.

We know that if the ball is on an incline, then it moves slower than in free-fall. And we can also imagine that a fall from a steep incline takes less time than the same fall from a not-so-steep incline. In fact, if we decrease the incline until it is horizontal then gravity is cancelled by the incline and the ball doesn’t move at all.

It makes sense to “time” the glider as it “falls” from different heights on an airtrack of fixed, small incline. The time should increase as the displacement increases. But by how much will the time increase for a given increase in displacement? This is the very relationship we choose to discover in this lab.

We will begin each “fall” at different heights (call it the Start Position). We will end each fall at the same position (call it the Final Position). The displacement is simply the difference between the two positions.

\[
\text{Displacement} = |\text{Final Position} - \text{Start Position}| \quad \text{Equation (1)}
\]

The absolute value bars make certain the displacement value is positive.

For each displacement we will measure the Time it takes to get from the Start Position to the Final Position. But to be certain we are consistent, we will drop it from a given Displacement three Times.

Plotting Displacement versus Time (Displacement (being first) is on the vertical axis) should help us determine how these two values are related.
Equipment

- Airtrack
- Glider
- Riser Block C (1 cm)
- Stopwatch
- Two Meter Stick (attached to the airtrack)
- Sticky-Note for Final Position marker.

Procedure

Procedural Synopsis

Level the Airtrack. Raise the single leg of the incline using Riser Block C (1 cm). Place a sticky note at the “bottom” of the track for a Final Position mark. Test the track’s incline.

Place the Glider at the first Start Position (0.20 meters from end of the track). Time its motion from rest until it reaches the Final Position marker. Repeat this same time measurement twice more for the same Start Position. Place the Glider at the next Start Position (10 cm further) and record its falling Time1,2, and 3. Repeat step 8 until you have completed timing all sixteen trials.

1. **Level the Airtrack.**
   a. Turn on the blower to half its maximum. (The blower knobs are sometimes misaligned with the numbers so you may need to find 1/2 max unconventionally.)
   b. To adjust the Airtrack you can adjust the side with double support feet by rotating them as you would a screw.
   c. Do this until the Glider is still. (Though a little movement is okay.)
   d. Please check to see that there is a bumper at each end of the Airtrack.

   **Caution**
   Please keep the blowers on for a minimum amount of time.

2. **Raise the single leg of the incline using Riser Block C (1 cm).**
   Measure the height of this block with the vernier caliper and record it as Height of Block above the data table.

3. **Place a sticky note at the “bottom” of the track for a Final Position mark.**
   a. Use the lower side of the track (the side with the double support legs).
   b. Place it 10 cm from the end of the track. Use the end with the blower input. Record this position in meters as the Final Position above the data table.

   **Caution**
   Please do not ever mark on or tape any of the lab equipment!

   **Caution**
   BE CERTAIN TO USE THE SIDE OF THE TRACK THAT HAS 0.0CM NEAR THE END WITH THE BLOWER INPUT!!

4. **Test the track’s incline.**
   a. Place the glider one meter away from the Final Position mark.
   b. Time its motion from rest until it coincides with the Final Position mark.
   c. Your time should be around 4.5 ±0.2 seconds.
      (i) If it is not, then perform steps 1 through 3 again but more carefully.
(ii) If after repeating the steps it still doesn’t work, then consult your instructor.

5. Place the Glider at the first Start Position (0.20 meters from end of the track).
   Place the front of the glider at this position. (It should be 10cm from the Final Position.)

6. Time its motion from rest until it reaches the Final Position marker.
   This is Time1 (s) and you should record it in the proper data column in the data table.
   
   Caution Please keep the blowers on for a minimum amount of time.

7. Repeat this same time measurement twice more for the same Start Position.
   These correspond with Time2 and Time3 (s) for Trial 1.

8. Place the Glider at the next Start Position (10 cm further) and record its falling Time1,2, and 3.
   ∅ Your Start Position for each trial should match that in the data table.

9. Repeat step 8 until you have completed timing all sixteen trials.
   You should be able to knock out all these measurements in < 7 minutes if you devise a shared method of data recording.
   a. Have one person in charge of the Glider.
   b. Have one person in charge of the Stopwatch.
   c. Have one person in charge of recording values.
   
   Caution Please keep the blowers on for a minimum amount of time.

Data Analysis

The analysis is lengthy with calculations. For each type of calculation you may want one person to start at Trial 1 and another to start at Trial 16.

10. Calculate the Average Time for each trial.
    ∅ As with all proper data tables, the description of this data column and its formula is located near the data table. This time it is below the data table under the obvious heading of Average Time (s).

11. Calculate the Displacement for each trial.
    This should be a positive value so use the absolute value for this calculated displacement.

12. Plot Displacement versus Time.
    ∅ Note that Displacement versus Time means to plot Displacement on the vertical axis and Time on the horizontal axis. When told to plot This versus That, This always belongs on the vertical axis and That belongs on the horizontal.
    
    Caution Do not connect the dots!
Remember that we are trying to discover how Displacement and Time are related.

You can tell from your plot if Displacement and Time are directly proportional to each other if there exists a straight line that describes the data points. Pick up the graph paper and look (from the paper’s edge) down the origin at the data points. Does it look like a straight line or a curve would better describe the data?

Look at the Time interval between two and three seconds. How much did the Displacement increase? _______

Look at the Time interval between four and five seconds. How much did the Displacement increase? _______

In other words: For each increase in Time by one unit, does Displacement increase regularly?

Hopefully your conclusion is that Displacement and Time are not directly proportional. (There exists no straight line that describes the entire data range.)

When two variable are proportional to each other, it means that if you double one, the other one will double as well. If you triple one, the other triples. And so on and so forth. This is proportionality between variables. Displacement and Time do not share this.

What mathematical function could you perform on Time to make Displacement increase regularly for each unit of this new Function(Time). In other words: What math function could straighten this data out into a line.

As you saw from looking at the two Time intervals, Displacement is increasing “faster”. You could leave Displacement the same and make the horizontal axis “increase faster” by squaring its value (Time). You have not changed measured data by doing this; instead, you will create a new plot to display this new relationship. The plot is Displacement versus Time^2.

13. Calculate the (Average Time)^2 for each trial.

14. Plot Displacement versus (Average Time)^2.

Hopefully your plot of Displacement versus Time^2 looks linear when viewed from the paper’s edge, peering down from the origin.

15. Draw a best-fit line that properly describes the data’s trend.

Caution

Do not draw a line that connects the dots!
Instead draw a line that has equal weight above it as below it.
The line doesn’t even have to touch one single data point!

Hold the graph paper up and look down the page from the origin. You should see a linear trend in the data. Use a straight edge to draw this best-fit line. Do Not connect the data points with the line. A proper best-fit line should have equal weight above as below the data. Do Not force the line to go through the origin!
16. Find the slope and intercept of the best-fit line.

Notice in the figure that the best-fit line travels through the grid lines at least twice. Use these intersections for the rise/run calculation.

Do Not use data points to calculate slope!

Estimate the value for the vertical intercept.

Find the slope of your line (Not the data!) by finding two places where the line goes through grid lines. Mark those places distinctly and write their coordinates next to the marks. Do the math for the slope directly on the graph paper somewhere.

Find the vertical intercept for your line by drawing it straight back to the vertical axis. Estimate the value as best as you can.

These Next Two Steps are Critical.

17. Report the equation of the line on the graph in y=mx+b format.
   Replace the m and b with numbers. See the figure above.

18. Rewrite the line’s equation on the graph.
   This time replace the y and x with their proper names and include units for the values of m and b. See the figure above.
Data And Graphs

Lab Station Number:

Names Sorted Alphabetically (by last names)

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Date Data Was Taken:

Instructor's Name:

Lab Time:

Examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):

DATA TABLE
DISTANCE VERSUS TIME
DISTANCE VERSUS TIME^2
ANALYSIS

Properly Finish This CoverSheet (5pts)
## Data and Graphs: An Inclined Plane:

### Constants:

<table>
<thead>
<tr>
<th></th>
<th>Measurement Uncertainty</th>
<th>Relative Uncertainty</th>
<th>Uncertainty Method</th>
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</thead>
<tbody>
<tr>
<td>Final Position</td>
<td>2.100 [m]</td>
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<tr>
<td>Height of Block</td>
<td></td>
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<td>Distance b/w Legs</td>
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<td>Sin (θ) = H/L</td>
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</table>

### Measurement Uncertainty:

- 1 pt measurement with motion and parallax
- Resolution of Vernier Caliper
- 2 pt measurement with parallax
- Sum of Relative Uncertainties

### Table:

<table>
<thead>
<tr>
<th>Trial</th>
<th>Start Position [m]</th>
<th>Time1 [s]</th>
<th>Time2 [s]</th>
<th>Time3 [s]</th>
<th>Average Time [s]</th>
<th>σ_{avg t} [s]</th>
<th>(Avg. Time)^2 [s^2]</th>
<th>Displacement [m]</th>
<th>Theoretical Distance [m]</th>
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### Calculations:

- \( m_{predicted} = \)
Table Definitions:

<table>
<thead>
<tr>
<th>Column Titles</th>
<th>Explanation / Formula</th>
<th>Calculation: Trial One</th>
<th>Value: Trial One</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start Position [m] =</td>
<td>Start Position as read on the airtrack</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>time1, 2, 3 [s] =</td>
<td>time to &quot;fall&quot; from Start Position to Final Position</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Average time [s] =</td>
<td>(t1 + t2 + t3) / 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>σavg t [s] =</td>
<td>sqrt(((Avgt-t1)^2+(Avgt-t2)^2+(Avgt-t3)^2)/(3-1))</td>
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<td></td>
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<tr>
<td>(Avg. time)^2 [s^2] =</td>
<td>Average time * Average time</td>
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<tr>
<td>Distance [m] =</td>
<td></td>
<td>Final Position - Start Position</td>
<td></td>
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<tr>
<td>Theoretical Distance [m] =</td>
<td>? For week 2</td>
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ANALYSIS

The glider travelled down an incline. When objects travel down an incline, we expect acceleration to occur. For accelerating objects, how does displacement relate to time? Recall that objects undergoing constant acceleration have the following relationship between displacement and time:

\[ \text{displacement} = d - d_0 = \frac{1}{2}at^2 + v_0t, \]

where \( a \) is acceleration (a constant in this case) and \( v_0 \) is the initial velocity.

1. For this experiment, what is the initial velocity of the glider? ________

You just finished plotting displacement versus time squared. According to Equation 1 the slope of your plot must be equal to \((1/2)a\).

2. State the value of the slope (don’t forget to include the units.) ___________

3. Does \( a \) have the same units as your slope? _______

Therefore,

\[ \text{slope} = \frac{1}{2}a. \]

Equation(2)

4. What is causing this acceleration? ________

It is due to gravity. Now, let’s just say (for the time being) that \( a = g \). (Where \( g \) = acceleration due to gravity = \( 9.8 \text{ m/s}^2 \)).

5. Then what should be the predicted value of the slope? _______________

(Show your work!)

6. Compare the predicted value to your value of the measured slope. Or, is your value for the slope relatively close (say within 20%) of the predicted slope? (Show your work.)

\[ \sin \theta = \frac{H}{L} \]

Hopefully, you found that the predicted slope is much higher than the measured slope. This result indicates that the direction of gravity is not the same as the incline (the incline is not vertical.) In fact, the component of gravity that is directed along the incline depends on the angle of the incline, \( \theta \). When the incline is horizontal, \( \theta=0^\circ \) and \( a=0 \). When the incline is vertical, \( \theta=90^\circ \), then \( a = g \).

The trigonometric function sine performs this task quite nicely. In general, the component of \( g \) that is parallel to the incline is \( g|| = g \sin \theta \). Note that Sin (\( \theta \)) is dimensionless, so \( g \sin \theta \) has units the same as \( g \).
Finally, then, the predicted slope is

\[ \text{slope} = \frac{m_{\text{predicted}}}{2g \sin \theta} \quad \text{Equation (3)} \]

7. Determine the value of \( \sin \theta \) for your experiment.

\[ \sin \theta = \frac{H}{L} \quad \text{Equation (4)} \]

8. Determine the value of the predicted slope. (Use \( g = 9.8 \text{m/s}^2 \)).

\[ m_{\text{predicted}} = \frac{1}{2} g \sin \theta \quad \text{Equation (5)} \]

9. What is the measured slope (from your data)?

Measured Slope from your Plot = \( m_{\text{data}} = \) \( \text{Eqn. (6)} \)

10. Compare your measured slope to the predicted slope by using

Relative Difference = \( \left| \frac{m_{\text{data}} - m_{\text{predicted}}}{m_{\text{predicted}}} \right| \times 100\% \). Is it within 10-20%?

(Show your work.)

We can usually compare the two slopes by plotting the predicted curve. We can predict the displacement as a function of time.

\[ \text{Displacement}_{\text{predicted}} = \left( \frac{1}{2} g \sin \theta \right) t^2 \quad \text{Equation (7)} \]

11. You have an extra column in the data called \( \text{Predicted Displacement} \). But only one value is needed to finish the predicted “curve”, assuming the other value is (0,0).

a. Calculate the \( \text{Predicted Displacement} \) value for your longest trial of the data table. Use your time value; use \( g = 9.8 \text{m/s}^2 \); and use your value for \( \sin \theta \) that you found in Equation 4.

\[ Time^2 \text{ for longest trial} = \] \[ \text{Predicted Displacement for longest trial} = \] (Show your work.)

b. Now, what is the \( \text{Predicted Displacement} \) for \( t = 0 \) sec?

(It should be 0m).

c. Take these two data points (the one for the longest trial and for \( t = 0 \)) and use them to draw a straight line on your plot. This straight line is the line of \( \text{Predicted Displacement} \). Label it as such. Write its equation next to the line. (Use the predicted slope with its units for the equation that your write.)
Hopefully you have found that the predicted slope is consistent with your measured slope. You can know they are consistent by looking at Equation 5 and Equation 6 or simply by looking at the plot and seeing the lines are near to being parallel.

Historically (e.g. Galileo) this measured slope was used to determine $g$.

12. By equating your measured slope to $1/2gsin\theta$, solve for $g_{calculated}$ and give units.
(Show your work.)

**Error Analysis:**
The accepted value for $g$ is $9.8\text{m/s}^2$.

13. What is the relative error in your calculated value for $g$?

$$\text{Relative Error} = \frac{|9.8-g_{calculated}|}{9.8} \times 100\%$$  
Equation (8)

**Is my $g$ consistent with the accepted $g$?**
(?) within the confines of my experimental uncertainties ?)

What defines the size of the measurement uncertainties in Toto?

* in toto Latin [*ɪn ˈtɔtəʊ*] *

$$D = \frac{1}{2} g \sin \theta \ t^2 = \frac{1}{2} g \frac{H}{L} \ t^2$$

Each of these measured values has uncertainty.

We use $D$ and $t^2$ to find slope. Then we use $H$ and $L$ with the slope to find our value for $g$.

The acceleration due to gravity, $g$, is froth with uncertainties.

**We have uncertainty in each measurement.**

Let’s call the uncertainty in Height $\delta_H$; where $\delta$ means some small value.

The relative uncertainty in Height is

$$\frac{\delta_H}{H} \times 100\%$$

The relative uncertainty in Displacement is

$$\frac{\delta_D}{D} \times 100\%$$

The relative uncertainty in $L$ is

$$\frac{\delta_L}{L} \times 100\%$$

The relative uncertainty in Time is

$$\frac{\delta_t}{t} \times 100\%$$
**Error Propagation**

What is error propagation?

A question in error propagation is that when we take a product of measurements we do what with the uncertainties? Should our uncertainties get bigger or smaller as they propagate through the formulas?

Take a square and measure one side. What happens to the uncertainties when you calculate Area?

Can this be beneficial when our product contains measurements of different units?

The rule is to find the relative uncertainty in a product of measurements simply add the relative uncertainties.

\[
\frac{\delta g}{g} = \frac{\delta H}{H} + \frac{\delta L}{L} + \frac{\delta D}{D} + 2\frac{\delta t}{t}
\]

But why the 2 in front of the \( \frac{\delta t}{t} \)? Also, notice I have suppressed the 100%. And the result is not the uncertainty in \( g \), but the relative uncertainty in \( g \). How do we find the uncertainty in \( g \)?

\( \delta h \) (comes from resolution of vernier caliper. 0.00005[m])
\( \delta L \) (comes from 2pt measurement with resolution uncertainty.)
\( \delta D \) (comes from 2pt measurement with parallax and motion.)
\( \delta t \) (comes from (cheap) Standard Deviation of a trial.)

For the time uncertainty either use Standard Deviation from one trial’s \( t_1,t_2,t_3 \); or use \( (t_{\text{max}} - t_{\text{min}})/2 \) as the uncertainty for one particular trial.
Now, build the values for
\[
\frac{\delta_H}{H}, \frac{\delta_L}{L}, \frac{\delta_D}{D}, \text{and} \frac{2\delta_t}{t}
\]

\[
\frac{\delta_H}{H}
\]

\[
\frac{\delta_L}{L}
\]

\[
\frac{\delta_D}{D}
\]

\[
\frac{2\delta_t}{t}
\]

\[
\frac{\delta_g}{g} = \frac{\delta_H}{H} + \frac{\delta_L}{L} + \frac{\delta_D}{D} + \frac{2\delta_t}{t} =
\]

State again the relative error between your g and the accepted g (Q13 above.)

Now, is my g consistent with the accepted g within the parameters of my experimental uncertainties?
21. Which of these two uncertainties is most prevalent (the largest source of error) for the experiment?  
(Justify your answer.)

22. With infinite resources, how could you reduce the error (improve the accuracy of the experiment)? What would the experiment be like?

23. In step 13 you found a relative error for your calculated $g$. How does this relative error compare to the uncertainty in your time measurement? (Is it the prevalent source of error?)  
(State both uncertainties and your observation.)

24. Based on your relative uncertainties, is the calculated value for $g$ consistent with the predicted (accepted) value for $g$? Explain.

25. If no, then briefly describe an experiment that would determine how important a physical process is to affecting the outcome.

26. It is vital that your plot have the following things. (If you were following directions, this should already all be done.)

   Your final analysis plot (as with each lab all semester) should have
   a. Axes properly labelled with units.
   b. Data Points with a best-fit (trend) line draw back to intercept the vertical-axis.
   c. A physics equation describing the trend line. The variables should be spelled out (or greek symbols for some labs) and the numbers need units within square brackets [ ].
      For example: Force = 3.123 [N/m] Stretch + 0.023 [N]
   d. A Comparative Line (or theory line) that goes through the origin.
   e. The comparative line also has a physics equation.
1D Motion

Metro State Physics / UCDenver Physics

February 13, 2009

1 EQUIPMENT NEEDED

- Computer, SW750 Interface, DataStudio Software
- Data Template: 1DMotion.ds
- Motion Sensor
- 2.2[m] Dynamics Track
- Dynamics Cart
- Wooden Block

2 SET-UP

2.1 Prepare Hardware/Software

First, turn on the black interface using the power-switch on the back right-hand side of the interface. Please do not rotate nor move the interface.

Find the ClassSupport folder on the desktop or in the Dock and find Lab1/One-1DMotion/1DMotion.ds or something like that.

Open that template file.

2.2 Calibrate Motion Sensor

We don’t so much need to calibrate the motion sensor for this 1DMotion experiment; but calibration is good practice for later labs.

Position the cart 1.1[m] from front of motion sensor screen. Use a 2[m] stick to carefully measure this distance. Don’t use the end of the stick!

Make sure the motion sensor is pointed directly down the track (with a very slight upwards direction {less than 5°}) and not pointed to the side or down.

Remove 2[m] stick and all obstructions.

Click the “Set Up” button near the top toolbar of DataStudio.
See picture below for the following steps.

1. Choose the Motion Sensor tab in the lower part the dialog screen.

2. Enter the “Standard Distance” as 1.1 [m].

3. Click the “Set Sensor Distance” button.

4. Then “Present Sensor Distance” should exactly match the “Standard Distance” you entered. Verify “Present Sensor Distance” correctly measures cart at different locations along track.

5. Very important: follow step 5 in the picture below. You need to exit the “Motion Sensor” tab by click the tab that says “Measurements.”

In the picture below, “Set Sensor Distance” button had not yet been clicked, so the Present distance does not yet match the Standard distance.
3  PROCEDURE

3.1  Save your file!

After filling out the cover sheet, save your file. See the section near the end called: “Wrap It Up” for important file saving tips.

You might need to Unlock the Workbook in order to edit the cover sheet. Click on the workbook window, then choose Display/Show Tools, this unlocks the workbook for you to edit or destroy...

3.2  Workbook Page 2: To Move or Not to Move: Smart Tool

On workbook page 2 you will notice a “Smart Tool”. Turn on the smart tool and drag its center around.

![Smart Tool](image)

Drag center of tool until it aligns with data to investigate.

\[(x, y) = \text{time}[s], \text{pos}[m]\]

To turn off the smart tool, toggle the tool button in the toolbar. Use this button on all subsequent workbook pages to investigate the data present on the screen.

Study the motion so as to be able to move (or not) such that your trace copies exactly the black, bold trace line on the graph. Where (position) does the model trace-line start?

Try to move the cart (or not!) so that the cart’s motion-data-trace matches the black line in the graph. Use the Start button when you are ready. The software will countdown from 3 seconds and then begin recording. A point will be on the axis during the countdown allowing you to adjust to the proper start position. Notice the countdown clock next to the “Start” button.

Note: Sound reflection is from first reflective object; back of cart, hand, notebook, or whatever is first in the “way” of the signal that will first reflect the signal.

Don’t forget to type out a description of the motion in the textbox below the graph. Be both Qualitative and Quantitative.
3.3 Workbook Page 3: Steady Motion: Smart Tool Delta Mode

On workbook page 3 turn on the smart tool and instead of dragging the center, put your mouse near the corner and notice a hand appear with a $\Delta$ symbol. When dragged, this expands to display the rise and run or $\Delta y$ and $\Delta x$ for the selection you drag.

![Smart Tool Delta Mode](image)

Study the motion so as to be able to move (or not) such that your trace copies exactly the black, bold trace line on the graph. Where (position) does it start? Where does it go? When? etc.

Try to move the cart (or not!) so that the cart’s motion-data-trace matches the black line in the graph. You want to see if you can create a situation where you push the cart once and then the rest is “automatic” in matching the model line. Use the Start button when you are ready. The software will countdown from 3 seconds and then begin recording. A point will be on the axis during the countdown allowing you to adjust to the proper start position. Notice the countdown clock next to the "Start" button.

You might find it useful to double-click the “StopWatch” or “Timer” display. There you will find Start and Stop conditions for data recording. One might set the start condition as follows: Delayed Start: Data Measurement: Position, Ch 1&2 (m): Rises Above...: 0.59 [m].

![StopWatch](image)

Keep your best two or three runs. Delete all others that were not as good.
Include at the bottom of the workbook page a description of the motion; mention qualitative and quantitative information about the position, velocity and acceleration in your text description. For example:

*The position started at 0.5[m] and changed to 1.2[m] over a time of 8[s]. It’s velocity was constant and equal to 0.09[m/s]. There was no acceleration during the first three seconds. During the next 5 seconds...*

Write the equations of motion x and v for the model line.

### 3.4 Workbook Page 4: Collision: Stats Tool, Linear Fit Tool

On workbook page 4 find the Statistics Tool Σ. Toggle it to the on position. Notice a drop down selection next to the Σ button. It lets you turn on and off certain stats; it might be useful to turn OFF the “Mean” statistic for this particular data.

![Stats Tool](image)

Study the model (black line) motion. Where (position) does the motion begin? Where does the cart go next? How long does it take to get there? etc. Use the stats tool to find those values quickly. Use the smart tool to confirm points and to investigate velocity.

Set up a situation so that the cart’s motion-data-trace closely matches the black line in the graph. Try to make it so that you are not touching nor guiding the cart during the data recording. Don’t forget about the Start and Stop Conditions found in the StopWatch display; they might help you get clean data.

Include at the bottom of the workbook page a description of the motion; mention qualitative and quantitative information about the position, velocity and acceleration in your text description. What was the velocity before the collision? After? For example:

*The position started at 0.5[m] and changed to 1.2[m] over a time of 3[s]. It’s velocity was constant and equal to 0.23[m/s]. There was no acceleration during the first three seconds. During the next 5 seconds...*

What was special about the model that is not very realistic? Is it plausible to exactly match this model with your motion at the t=4[s] mark?

Further Tools to learn: Linear Fit Tool Find the Linear Fit option within the “Fit” tool.

![Linear Fit Tool](image)

Instantly it applies a fit to the entire data
set. All the stats, including Linear Fit, will adjust themselves according the data you select with the plain ole’ mouse. Notice the pictures below show the linear fit applied to the entire data set and the next picture shows a careful selection of data’s first part. This model data is difficult to select compared to real data because the model data only has a total of three data points. That is why in the selection below you will see a box selected slightly bigger than the desired data. The box selected in the picture below shows the model highlighted in yellow. The linear fit is applied to only that yellow-highlighted portion.

3.5 Workbook Page 5: Falling Up (or Down?) an Incline: Quadratic Fit Tool.

On workbook page 5 you will see motion that appears to have some sort of smooth change associated with it. What situation can you create to match this data?

Don’t record data yet.

What will you need to do in regard to position to match the data? Where (position) does it start, where does it end, how long does it take?

What is happening to velocity in this graph? (How does that compare to the previous graph?)

What will you need to do, in regards to acceleration, to match the data?

There is another powerful tool to help in discovering the equations of motion for this parabola. First choose “GDMatch” in the legend so that it is the active data set. Then find “Quadratic” in the Fit drop down menu.

Here it is important to note that the Fit, Smart, and some other tools are applied to single sets of data. Statistics automatically applies to all data visible. But these other tools have to be chosen, or turned off, after you choose a data set by selecting the data set in the legend.

When you turn on the Quadratic Fit, you will see a box with values in it for A, B, C, Mean Squared Error, and Root MSE. If you double click that output box, you will see a explanation dialog box open as pictured below. There you will discover the form of the fit equation and how A, B, and C are related to that equation.
A portion of the “Curve Fit” dialog box is shown above. In it you can see this expression: $Ax^2 + Bx + C$. It means that $y = Ax^2 + Bx + C$. Where $y$ = position and $x$ = time. So we can rewrite the equation to say this

$\text{Position} = A \text{time}^2 + B \text{time} + C$.

But what are the units? Position is measured in meters. Time is measured in seconds. But what about the constant $A$, $B$, and $C$?

$\text{Position}(t)[m] = A[\frac{m}{s^2}] \text{ time}^2 + B[\frac{m}{s}] \text{ time} + C[m]$.

So $A$ must be some sort of acceleration. $B$ must be some sort of velocity. And $C$ is some position value.

Remember from lecture the kinematics equation:

$y = y_o + v_o \ t + \frac{1}{2} a \ t^2$.

Where $y$ is the position at any point in time;

$y_o$ is the initial position (at time=$0[s]$);

$v_o$ is the initial velocity;

and $a$ is the constant acceleration required for kinematic motion.

Three kinematics equations are:

$y = y_o + v_o \ t + \frac{1}{2} a \ t^2$.

$v = v_o + at$

$a = constant$

Look at these two equations:

$\text{Position}(t)[m] = A[\frac{m}{s^2}] \text{ time}^2 + B[\frac{m}{s}] \text{ time} + C[m]$.

$y = y_o + v_o \ t + \frac{1}{2} a \ t^2$.

What is $A$? What is $B$? What is $C$?

$C = y_o$.

$B = v_o$.

$A = \frac{1}{2}a$.

Be careful with the acceleration; it is twice $A$.

So, what are the equations of motion based upon the quadratic output?
We know
\[ A = -0.0222 \text{ m}^2 \text{s}^{-2} = \frac{1}{2} a. \]
\[ B = 0.355 \text{ m} = v_o. \]
\[ C = 0.200 \text{ m} = y_o. \]

Therefore we can write the kinematic equation of motion as
\[ \text{Position}(t) = -0.0222 \text{ m}^2 \text{ time}^2 + 0.355 \text{ m} \text{ time} + 0.200 \text{ m}. \]

Remember three kinematics equations are:
\[ x = x_o + v_o t + \frac{1}{2} a t^2. \]
\[ v = v_o + at. \]
\[ a = \text{constant}. \]

Rewriting with constants yields:
\[ x = -0.0222 \text{ m}^2 \text{ time}^2 + 0.355 \text{ m} \text{ time} + 0.200 \text{ m}. \]
\[ v = 0.355 \text{ m} + -0.0444 \text{ m} \text{ t}. \]
\[ a = -0.0444 \text{ m}. \]

We have been using \( y \) and \( x \) for position; this is not to be confused with the \( x \) which is time, on the DataStudio graph.

Remember that \( a = g \sin(\theta) = g \frac{H}{L}. \)

Write all the equations of motion (of model line) below the graph.

Set up your situation (maybe an incline?) to match this motion. Keep your best trial and delete the “not so good” trials for this graph.

3.6 Workbook Page 6: SHM, Simple Harmonic Motion

This motion is actually the easiest to match! Since we were all babies, we were rocked back and forth; we’ve been on swings; maybe some of you have even bungee jumped!

The calculus students should be able to easily differentiate this equation of motion to find the first and second time derivatives \( v \) and \( a. \)

All students should apply a sine fit to the data using DataStudio; although, we wrote this function as a cosine instead of sine. (Sine and cosine are simply different from each-other with a \( \frac{\pi}{2} \) phase shift.)

\[ \text{Wiggle} = \text{amplitude} \times \cos(2 \times \pi \times \text{frequency} \times \text{time}) + \text{offset} \]
\[ y(t) = 0.4 \cos(2\pi \times \frac{1}{3} t) + 1.2. \]
\[ v(t) =? \]
\[ a(t) =? \]

Whether you are a calculus student or not, take a moment to draw the curves for velocity and acceleration on a piece of paper.

What do you notice about the acceleration? Is it constant?
What do you notice about the peak in acceleration?
Where does the peak acceleration occur on the position graph?
How about the peak velocity on the position graph?
Label all these on the graph.

On a swing, when does a small child get the biggest eyes and their diaphragm gets stuck so they can’t even scream? What does the inner ear sense?
4 Wrap It Up

Save the file to the Desktop using this naming convention SmithBarnesJones 1DMotion.ds Use your last names, the experiment title, _ instead of spaces, and end it with .ds.

Choose File/Page Setup and then choose the Landscape (rotated) Orientation. Choose File/Print and save a pdf, if desired. Use SmithBarnesJones 1DMotion.pdf for the file name. Make sure the pdf ends in an extension of .pdf instead of .ds.

Do not close windows as you do in that “other” operating system.

Quit Data Studio using the File/Quit menu.

Do this next step only after you have Quit DataStudio from the DataStudio Menu. Drag your data file to the desired location whether it be a USB drive or sftp location. Close the sftp session when finished. Right-click on the USB device and choose “put away” before you pull out USB drive.

Please don’t forget to turn off the interface box.

Log out of the computer.

Your group will hand in one write-up for the group. The Write-Up is the printout of the DataStudio Workbook. Please put all names on the front cover sheet. Alphabetize them but write them normally: John Borger, Mathew Smith, Sarah Zachariah.

5 PostLab

Ask your instructor or visit the web page for notes about the PostLab. It may be that this PostLab is NOT online, but rather some worksheets for you to fill out.
1DMotion

Lab Station Number: 

Names Sorted Alphabetically (by last names)

<table>
<thead>
<tr>
<th>First Name</th>
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Date Data Was Taken: 

Instructor's Name: 

Lab Time: 

Examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts): 

5 Pages with equations for x, v, and a:
The equations are meant to describe the model lines (curves), not the actual student data. It is perfectly acceptable to handwrite equations on the following pages...

No Motion ...thus, save paper.
Steady Motion The computer is needed
Collision to find the constants etc.
Falling Up for the equations.
SHM Then hand-writing is okay...

Properly Finish This CoverSheet (5pts)
Match This Motion! Then describe the "motion" in words with values too.

TO move OR NOT TO move
Match This Motion! Then describe the "motion" in words with values too. Include the equations of motion x and v for the model line.

Steady Motion
Match This Motion! Then describe the "motion" in words with values too. Write out equations for x and v.
Match This Motion! Then write out \( x \), \( v \), and \( a \) for the model line.

\[ \text{Falling Up} \]
Match this motion! Then describe in words about $x$, $v$, and $a$. 

SHM
1DMotion POSTLAB

Lab Station Number: 

Names Sorted Alphabetically (by last names)

<table>
<thead>
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Instructor's Name: 

Lab Time: 
examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):
The following pages are in this order:!
Collision2, Falling, and SHM (Simple Harmonic Motion.)

Instructions:
Label each axis with name and units,
and apply basic scale to each axis.
Write out all equations x, v, and a;
and don't forget units!!
Draw Curves for v and a.
Collision

\begin{align*}
x(t) &= \quad \quad \quad \quad \quad \quad \\
v(t) &= \quad \quad \quad \quad \quad \quad \\
a(t) &= \quad \quad \quad \quad \quad \quad 
\end{align*}
Falling Down

\[ y = Ax^2 + Bx + C; \]
\[ A=0.0444, B= -0.355, C=1.60 \]

- \( x(t) = \)
- \( v(t) = \)
- \( a(t) = \)
Simple Harmonic Motion (SHM)

\[ y = 0.4 \times \cos(2\pi \times \frac{1}{3} \times x) + 1.2 \]

\[ x(t) [m] = \]

\[ v(t) = \]

\[ a(t) = \]
# MSExcel-Spreadsheets

**Lab Station Number:**

## Names Sorted Alphabetically (by last names)

<table>
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For the Spreadsheet lab you will turn-in a digital file (NOT PAPER!) to be graded by your instructor. Your instructor will look at your cells for equations and look at your plots. (No! Not your human cells, but your spreadsheet cells!)

The tutorial begins with a spreadsheet template (an MSExcel file.) Then you watch movies. After each movie, perform the same steps in your template file. When finished, save your file and show your instructor the file on one of the lab computers or email the file to your instructor.

If you email the file: within the email include your name, the classtime, and whatever else is normally on the coversheet for Write-Ups.

You can find the files on the DVD (or in the near future, online via youtube or something similar: Douglas Howey is waiting to hear from students... which is better, online or dvd...)

Be sure to pick the latest version of MSExcel tutorials!
Atwood’s Machine

MSCD/UCD Physics

May of 2008

EQUIPMENT NEEDED

- Computer, SW750 Interface, DataStudio Software
- DataStudio Template: AtwoodTemplate.ds
- DataTable.xls
- Table Clamp
- “Smart” Pulley
- Braided Massless String
- Two (high dollar value) mass hangers
- HANGER1 + 1 WASHER + 50g + 20g + 6 WASHERS.
- HANGER2 + 2 WASHERS + 100g.
- Mass Balance
PURPOSE

This lab is to study the relationship between force, mass, and acceleration using an Atwood’s Machine apparatus. In an Atwood’s Machine, the difference in weight between the two hanging masses determines the net applied force acting on the system. This net force accelerates both of the hanging masses.

THEORY

If the pulley can be neglected, the tensions in the strings are internal to the system, and will cancel each other. Therefore the net applied force will be the differences in the masses multiplied by the acceleration due to gravity. Then Newton’s Second Law can be used to find a relationship between the masses and the expected acceleration, \( a \).

Total Mass \( \times \) Acceleration = Difference in Mass \( \times \) Gravity. If we add in the friction from the bearing we can say

\[
\Delta mg = \Sigma ma + 2f
\]

See Figure 1 on Page 3 for a more thorough development of theory.

Saving Files is Important

It is important to save your file to the Desktop. Do *not* save your file to any place except the Desktop. Especially do not save to a usb flash drive. After the experiment is finished you may save again and quit. Then drag the file from Desktop to your usb stick.

Give the file a good name such as “AugustaWindAtwood.ds” Don’t use spaces; and be sure to end with “.ds” so that the file can be opened on another computer. Save often during the period so as not to forfeit your work to a stray electron.

Save the .xls file also.

PROCEDURE

SetUp

Movie: 01 Procedure.mov

Set up the clamp and pulley/smartpulley system at the edge of the table. Use a piece of thread about 10 cm longer than the distance from the top of the pulley to the floor. You can fasten the mass hangers to the thread using a tied loop.

Turn on the interface. Find and open the DataStudio template. Also find and open the spreadsheet template.

Create a hanger system \( m1 \) of about 110g:

\[
\text{Mass1} = \text{Hanger1} + 1 \text{ Washer} + 50g + 20g + 6 \text{ Washers}
\]

(Yes, put them on in this order.)

Create a hanger system \( m2 \) of about 115g:

\[
\text{Mass2} = \text{Hanger2} + 2 \text{ Washers} + 100g.
\]

(Yes, put them on in this order.)
Imagine the two masses being lifted up so that all the forces are in a line:
Now, start from the left (negative direction); and add the forces. Newton’s Second law says that the sum of these forces should be equal to the product of \( \text{system mass, acceleration} \).

\[
\Sigma F = M_1a = (m_2+m_1)a = -m_1g + T - f - f - T + m_2g \\
\Sigma F = (m_2+m_1)a = \Sigma m a = (m_2-m_1)g - 2f \\
\Sigma F = \Sigma ma = \Delta mg - 2f
\]

**Sum of the Forces = NSL = Applied Force - Friction**
The system of masses will experience an acceleration. This acceleration comes from the applied (driving) force made from the two masses being different in mass from one another. But this acceleration is going to be offset by the friction in the bearings. Let’s rewrite the equation:

\[
\Delta mg = \Sigma ma + 2f
\]
Be certain to measure each mass system and enter the two values in the spreadsheet data table. Enter them as kg NOT as grams!

Movie: 02 Velocity vs Time.
Create a Plot of Velocity versus Time.

**Data Taking**

Move the heavier of the two masses upward until the smaller mass almost touches the floor. Click the “Start” button and release the mass. The data will start when one of the spokes of the pulley rotates to intercept the photodiode. The recording will stop automatically after three seconds. If not, then click “Stop.”

Movie: 03 NameRunsAsValues
Rename the DataRun to match mass values (i.e. name the data run “111.3g 115.8g” because those were the values of your two masses.)

Movie: 04 Acceleration
Find the Acceleration by using a linear fit upon the whole portion of the data that represents the dropping mass. Do this by using mouse to select portion of graph showing relatively constant acceleration. Turn on “Fit/Linear” and the slope of the line will be calculated. The slope is equal to the acceleration.
acceleration = change in velocity / change in time.
Record acceleration value in the data table.

FOLLOW THIS CAREFULLY!
Now you need to archive the graph in MSExcel, so that you can have a “Sample Run” for your WriteUp.

But FIRST, you need to add an annotation. Use the Annotation Tool “A” to add a text box where you will discuss what is being plotted and how you know the acceleration value.

Then choose Edit/Copy and switch to the spreadsheet and Edit/Paste Special the picture into the spreadsheet on the worksheet called “Sample Data.”

**Continue Data Taking**

When you have successfully archived the graph, get rid of the annotation, or Create A New Graph to do the rest of the experiment.

You only need to archive and hand in one sample graph from DataStudio.

Movie: 05 Graph Maintenance
You may Clear the Data Run from the Graph (but don’t delete it!)

Shift approximately 5 grams of mass (1 washer) from the lighter mass system to the heavier mass system. This allows you to change the net applied force without changing the total system mass. What do you suppose will happen to the acceleration of the system?

Record another run of data with the new hanger systems.

Mass each hanger system each time you change system masses.

Be certain to enter the values in the data table.

Don’t forget to rename each run in DataStudio, as you record them! (This will possibly save you a lot of time later on, if you have goofed up the analysis some how...)

Repeat these steps until the data table is filled.
UNCERTAINTY Data Collection

After you are finished, you need to establish what measurements you took and their uncertainties.

For the Acceleration data (Yes, we call this a measurement.) you need to pick the last mass system (or second to last) and repeat the drop 5 times. Rename these runs something like this: Uncertainty1, Uncertainty2, ...

Record each acceleration you measure into the data table near the bottom; find average and standard deviation. This standard deviation is the uncertainty in the acceleration measurement found by repetitive measurements. You will report this average and uncertainty on the PostLab.

Investigate Friction

Do one more run like this:
Turn OFF the Automatic Stop condition (Double-click the stopwatch and set the Automatic Stop Condition to none.)
Then just spin the SmartPulley and click “Start.”
Stop collecting after a while or when the pulley nearly stops moving. On the web site I have some pictures of data runs where I set up the pulley near the ceiling and had a really long string to investigate friction and string mass shifting from one side to the other.

ANALYSIS

Create the appropriate calculations in the data table for
+ total mass for each run
+ then an average total mass below that\(^1\)
+ Applied Force (difference in masses * acceleration due to gravity.)
+ NewtonIIforce (average system mass * acceleration.)

Make a graph from the data table of AppliedForce versus \(a\).

Also on the same graph, plot the theoretical line. (This has the expected mass in it.) The theory says that the Applied Force should equal Newtons Second Law. So Newtons Second Law is considered the theoretical data. Use the average total system mass (so that it is a constant) and multiply that by each acceleration. This is what NSL predicts.\(^2\)

Add a trend-line for the Applied Force data series.
Create a Physics Interpretation of that trend-line. And create a Physics Translation of the NSL data set.

Turn in the cover sheet, data table (print out), sample data run from trial 1, printout of Force vs. Acceleration Graph.

\(^1\)The system mass doesn’t change; but the column for \(\Sigma M\) will change due to measurement uncertainty. It is a good idea to add a cell to the right of Average System Mass and use =stdev(range reference for \(\Sigma M\) values).

\(^2\)You should not add a trend-line to this data set since it is theoretical. You already know the slope and intercept. Instead reformat the data set on the graph to remove the data points but show a dashed line.
Explanation of “Lost presence of mind.”

Subtitle: Atwood’s Machine for a Darwin Award Recipient.

This is a copy of an alleged letter written to an insurance company that wanted a complete explanation from an injured man who had explained what happened with the words “lost presence of mind.”

“I’m a bricklayer by trade. On the day of the accident, I was working alone on the roof of a new six story building. When I had completed my work, I discovered that I had about 500 pounds of bricks left over. Rather than carry the bricks down by wheelbarrow, I decided to lower them in a barrel by using a pulley which was attached to the side of the building at the sixth floor.

“Securing the rope at ground level, I went back up on the roof, swung the barrel out and loaded the bricks into it. Then I went back to the ground floor and untied the rope, holding it tightly to assure a slow descent of the 500 pounds of bricks. You will note in block 11 of the accident reporting form that I weigh 135 pounds.

“Due to my surprise at being jerked off the ground so suddenly, I lost my presence of mind and didn’t let go of the rope. Needless to say, I proceeded at a constantly changing rapid rate up the side of the building.

“In the vicinity of the third floor, I met the barrel coming down. This explains the fractured skull and broken collarbone. Slowed only slightly, I continued the rapid ascent, not stopping until the fingers of my right hand were two knuckles deep into the pulley. This explains the lacerations of my right hand. Fortunately, by this time I had regained my presence of mind and held tightly to the rope in spite of the pain.

“At approximately the same time, however, the barrel of bricks hit the ground, and the bottom fell out of the barrel. Devoid of the weight of the bricks, the barrel now weighed about 50 pounds. I refer you again to my weight of 135 pounds, in block 11. As you can imagine, I began a rapid descent down the side of the building. Again, to my surprise, my rapid rate increased constantly!

“In the vicinity of the third floor, I met the barrel coming up. This accounts for the two fractured ankles and the lacerations to my legs. The encounter with the barrel slowed me enough to lessen my injuries when I fell into the pile of bricks, and fortunately only three vertebrae were cracked.

“I am sorry to report, however, that as I lay there in the bricks, in pain, unable to move, and watching the barrel six stories above me, I again lost presence of mind; I let go of the rope.”

---

3Modified from here and many other online places: http://www.getamused.com/jokes/0121083.html
Atwood's Machine

Lab Station Number:

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Date Data Was Taken:

Instructor's Name:

Lab Time:

Examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):

Data Table
Annotated Sample Picture of Run 1 (Velocity vs. time)
Plot of Forces vs. Acceleration

Properly Finish This CoverSheet (5pts)
### Atwood’s Machine

**Data Table**

#### Names:

**Constant Total Mass:** The Mass is shifted from one hanger to the other.

<table>
<thead>
<tr>
<th>Column Name</th>
<th>Formula or Explanation</th>
<th>Sample Calc</th>
<th>Trial 1 Value</th>
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<tbody>
<tr>
<td>Mass_1 and 2 (kg)</td>
<td>Total Mass 1, Total Mass 2</td>
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<tr>
<td>Total Mass (kg)</td>
<td>?</td>
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<td></td>
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<tr>
<td>Acceleration (m/s^2)</td>
<td>Vel. vs. Time Slope (See SampleData)</td>
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<tr>
<td>Free Body Applied Force</td>
<td>( \Delta M \cdot g = (M_2 - M_1) \cdot 9.8 )</td>
<td></td>
<td></td>
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<tr>
<td>Newton II Force</td>
<td>?</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Trial #</th>
<th>Mass_1 (kg)</th>
<th>Mass_2 (kg)</th>
<th>Total Mass (\Sigma M) (kg)</th>
<th>Acceleration (m/s^2)</th>
<th>Free Body Applied Force (N)</th>
<th>Newton II Force (N)</th>
</tr>
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The Total Mass was semi-constant,  
\[ \text{Total Mass} = \Sigma M = \text{Average}(M_1 + M_2) = 0.000 \text{ kg} \]

(Use this value of average total mass to calculate NewtonII Force for each acceleration.)

**Uncertainty in Acceleration?**

Various Accelerations for the last set of masses:

<table>
<thead>
<tr>
<th>Accel.#1</th>
<th>Accel.#2</th>
<th>Accel.#3</th>
<th>Accel.#4</th>
<th>Accel.#5</th>
</tr>
</thead>
</table>

**Average** Choose Insert/Function and find the Average Function.

**Standard Deviation**

Choose Insert/Function and find The Standard Deviation Function.
Momentum

MSCD/UCD Physics

May of 2008

EQUIPMENT NEEDED

• Computer, SW750 Interface, DataStudio Software
• DataStudio Template: Momentum.ds
• Dynamics Track
• Two Motion Sensors
• Two Dynamics Carts, one with only one side of magnets
• 500 g mass

PURPOSE

To observe Momentum, the exchange of Momentum, and Impulse.

THEORY

Momentum is a vector equal to the product of mass and velocity. There are elastic collisions and inelastic collisions. Impulse is the change in momentum or the integral of the force versus time curve.

Perfectly elastic collisions do not allow the objects to decrease in kinetic energy. Inelastic collisions allow some kinetic energy to be transformed into other forms of energy. You might well guess that macroscopic systems cannot have perfectly elastic collisions. Conservation holds true. In a perfectly elastic collision both momentum and kinetic energy are conserved. In an inelastic collision only system momentum is conserved.

Newton’s Second Law can be stated as: \( \text{Force}_{\text{average}} = ma_{\text{average}} = \frac{\Delta mv}{\Delta t} = \frac{\Delta p}{\Delta t} \)

Impulse of Force is the product of average Force and the time interval.

\( \text{Impulse} = \text{Force}_{\text{average}} \Delta t = \Delta p \)

For Calculus students: \( \text{Impulse} = \int_{t_i}^{t_f} F(t) dt = \text{area under the Force vs. time curve.} \)
Saving Files is Important

It is important to save your file to the Desktop. Do *not* save your file to any place except the Desktop. Especially do not save to a usb flash drive. After the experiment is finished you may save again and quit. Then drag the file from Desktop to your usb stick.

Give the file a good name such as “ConstanceMomentum.ds” Don’t use spaces; and be sure to end with “.ds” so that the file can be opened on another computer. Save often during the period so as not to forfeit your work to a neutrino.

PROCEDURE

SetUp

0) Calibrate each motion sensor. (One is already set for negative velocity within the momentum calculation.) See your instructor if you don’t know how to calibrate well.

1) See the cart labels underneath the carts.
Cart One is closest to the computer. It has magnets on both ends.
Cart Two is furthest from computer. It has magnets on one side.

2) Level the track using constant motion or bubble level as your guide.

3) You must configure the calculations to match the mass of each cart at your station. The mass is recorded on the bottom of the carts.
Open the Calculator (next to stopwatch) and change the Experiment Constants (bottom third of calculator window) for mass1 and mass2 to match that of the carts. While you’re there input the value for the ExtraMass as recorded on the black mass. Use kg for the unit of measure. For each mass1, mass2, and ExtraMass use the “Accept” button in the bottom section of the Calculator. (Do not use the “Accept” in the upper right part of the Calculator.)

Please do not switch carts nor mass with any other table.

Data and Analysis

Follow the instructions in the DataStudio Template. Answers the questions and do the analysis as instructed there.
Momentum

Lab Station Number:

Names Sorted Alphabetically (by last names)

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Date Data Was Taken:

Instructor's Name:

Lab Time:

Examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):

Instructors are encouraged to save paper and grade this in class on the computers or as an email attachment, or...
The pages in the lab manual are in there so that students can sketch the graphs and then they can write the answers and even annotate the sketches.
A write-up done by hand, in this case, is perfectly acceptable, and it saves paper!
**Momentum = Mass * Velocity**

If an object's **velocity** doubles in size, what happens to the **momentum**?

If an object's mass doubles in size, what happens to the **momentum**?

Does **momentum** have direction?

If mass is measured in kg (kilograms) and velocity is measured in m/s (meters/second), then what units are used to measure **momentum**?

What would be an equivalent unit (more or less complex) for momentum?
For this first run record a run with the carts where Cart1 is moving and Cart2 is at rest near the center of the track. Stop recording a second or two after the collision. You want the gliders to gently bounce without sticking together; so magnets should be facing.
Questions:

What was the momentum of Cart1 just before the collision? Cart 2?

How about after for Cart1? Cart 2?

Did all the momentum switch from cart 1 to cart 2? Why or why not?

What is the difference between elastic and inelastic collisions?

Your Answers:


For this second run record the carts where Cart1 is stationary near the center of the track and Cart2 + Extra Mass are moving. Stop recording a second or two after the collision. You want the gliders to gently bounce without sticking together; so magnets should be facing.
What was the momentum of Cart1 just before the collision? Cart 2?

How about after for Cart1? Cart 2?

What is the sum of Momentum1 and Momentum2 prior to the collision?

What is the sum of Momentum1 and Momentum2 post collision?

What happened to the Momentum in Cart1? Cart2? Was there conservation of momentum?

**3rd Run: Cart1 moving, Cart2 at rest. Stick.**

For this **third** run record the carts where Cart1 is moving and Cart2 is at rest near the center of the track. **Take off the Extra Mass!** Stop recording a second or two after the collision. **You want** the gliders to gently stick together; so Cart2 should be turned around.
What was the momentum of Cart1 just before the collision? Cart 2?

How about momentum after for the system?

How do pre and post momentum compare for cart 1?

What is the sum of Momentum1 and Momentum2 post collision?

Are carts 1 and 2 reading the same momentum after the collision? Why or why not?

3rd Run: Cart1 moving, Cart2 at rest. Stick.
4th Run: Cart1 moving, Cart2 moving. STICK TOGETHER!

For this fourth run record the carts where Cart1 is moving and Cart2 is moving opposite so that they collide near the center of the track. Stop recording a second or two after the collision. You want the gliders to gently stick together; so magnets should be opposite sides of cart. Try to get carts moving at similar speeds!!!!
What was the momentum of Cart1 just before the collision? Cart 2?

How about after for Cart1? Cart 2?

What is the sum of Momentum1 and Momentum2 prior to the collision?

What is the system momentum post collision?

Where did the Momentum go???? Or was there any to go anywhere????

4th Run: Cart1 moving, Cart2 moving. STICK TOGETHER!
Investigate the Impulse Force. The area of Force vs. Time "under" the collision equals the change in momentum.
Explore an idea you have about momentum.
During the Spring Semester of 2006 I received a disturbing phone call from my mother.

"Douglas, we were just in a train wreck on the Amtrack Train in Southern California. But be at ease; your Dad and I are okay. We didn't even feel the collision with the Semi Truck-Trailer that was sitting on the track. The Engine was in the back of the train pushing it. We were the in the car closest to the engine. There were five cars altogether. The front car had one person in it, an employee, a look out. He pulled the emergency brake and jumped into the car behind him. That front car was smashed together pretty badly. But nobody on any car was hurt. Why didn't we even feel the collision?" (Yes the train and truck stuck together after the collision.)

How would you answer this question?
EQUIPMENT NEEDED

• Computer, SW750 Interface, DataStudio Software
• Data Template: EnergyWorks.ds
• Motion Sensor
• 2.2[m] Dynamics Track
• Dynamics Cart
• Two Wooden Blocks (2x4)
• Extra Mass (Black Metal Block)
• Vernier Caliper
• 2-meter stick

THEORY

To assist in class discussion:

Kinetic Energy = \( \frac{1}{2}mv^2 \)

Potential Energy = \( mg \, h \) (Often called U.)

Total Mechanical Energy = \( U + KE \)

Work (in terms of energy) = \( \Delta KE \)

Work (in terms of force) = \( F \, d \)

\( \sin \theta \) (for an incline) = \( \frac{H}{L} \)

What are the differences between conservative and nonconservative forces? Nonconservative forces take energy out of a system or add energy to the system. Conservative forces allow the energy of the system to stay constant.

What are the types of energy being measured today? What are the types of energy that we won’t measure but might be able to measure with more sophisticated equipment?

CALIBRATE SENSOR

Calibration is critical for this lab! You need to perform a careful sensor calibration because we are relying on the position measurement in many calculations.

Position a block somewhere around 1.1[m] from front of motion sensor screen. Use a 2[m] stick to carefully measure this distance. Do *not* use either end of the stick.

Make sure motion sensor is pointed directly down the track (with a very slight upwards direction of less than 5°) and not pointed to side or down.
Remove 2[m] stick and all obstructions.
Click the “Set Up” button near the top toolbar of DataStudio.

See picture below for the following steps.

1. Choose Motion Sensor tab in lower part of Setup Window.
2. Enter “Standard Distance” as 1.1 [m] or whatever value you used.
3. Click “Set Sensor Distance” button.
4. Then “Present Sensor Distance” should exactly match “Standard Distance” you entered. Verify “Present Sensor Distance” correctly measures cart at different locations along track.
5. Very important: follow step 5 in picture below. You need to exit “Motion Sensor” tab by clicking tab that says “Measurements.”

In picture below, “Set Sensor Distance” button had not yet been clicked, so Present distance does not yet match Standard distance.

Saving File is Important
It is important to save your file to the Desktop. Do *not* save your file to any place except the Desktop. Especially do not save to a usb flash drive. After the experiment is finished you may save again and quit. Then drag the file from Desktop to your usb stick.

Give the file a good name such as “MaryJohnFrankEnergy.ds” Don’t use spaces and be sure to end with “.ds” so that the file can be opened on another computer. Save often during the period so as not to forfeit your work to a stray electron.

CREATE KE SENSOR
Create a new calculation called KE. Program calculator to “measure” Kinetic Energy.
Remember: You will use cart and extra block (black mass.)
Be careful to set properties correctly!
We want to measure KE in mJ, not Joule.

Watch the movie about creating Calculation Sensors or read the text below.
You will find “Calculate” button next to StopWatch display.
Click “New” in calculator window.

Type an equation like this
\[ \text{KineticEnergy} = \frac{1}{2} \times \text{mass} \times \text{velocity}^2 \times 1000. \]

Upon hitting RETURN or ACCEPT, a section will appear asking you to define variables.

Drag velocity sensor from Summary Window and drop sensor on text that might say “Please define variable velocity.”

For mass, you can do one of two things.
A) Use drop-down menu next to “Please define variable mass” and choose “Constant.” Then type in mass value in kg.
Or B) Create an experimental constant in bottom portion of calculator. Be sure to click “Accept” in lower section of calculator in order to store your constant for use by any equation. Then choose drop-down arrow next to “Please define variable mass” and choose “Experiment Constant...”

Don’t neglect to go into “Properties” window and carefully set following values:
- Measurement Name → Kinetic Energy
- Variable Name → KE
- Units → mJ
- Accuracy → 0.1
- Precision → 1.

WORKBOOK PAGE 1
You will be recording your work in a digital workbook within DataStudio.

It is vital that you do not resize workbook window. It is preset to allow proper printing. If you make window too big, DataStudio will be forced to shrink everything upon printing.

Watch movie about Using Workbooks, if you need to have better explanation of using Workbook.

Don’t forget to “Display: Show Tools” now and next time you open file. Click workbook to make it active, then choose Display/Show Tools.

You can use tool bar on right side of workbook to add text, graph displays, and some other things as well.

On workbook page one, create a text box and record your constants and KE calculation you used.

We want cart to have extra black block-mass sitting on it during experiment. Cart and black metal block might have their masses pre-measured and written on a piece of tape.

WORKBOOK PAGE 2: Draw Free-Body Diagrams
This cannot be accomplished in DataStudio unless you can make a graphic in MSExcel, take a screen shot, convert it to .jpg using Preview and then import into DataStudio. So, don’t worry about drawing a digital picture! Draw one by hand.

Sketch two pictures by hand and simply insert this here (after page 1) after printing your final document.

One diagram is for forces while traveling up the incline. Other picture is for forces while going down the incline (but draw second picture, only if you think the two pictures are different.)

Include a description of differences, if any. Include an equation for \( \Delta F = ? \) where \( \Delta F \) represents the difference in the two NetForces.

WORKBOOK PAGE 3: Equation for Potential Energy U?
You are going to create an incline using wooden blocks. Position Sensor will be at top of incline. Put a system of two wooden blocks under legs of track that are nearest Motion Sensor. Height of blocks should be about 7-8 cm high.
In order to find \( \sin \theta \) you need to know \( H \) and \( L \). Use vernier to measure three heights from top of track to table (See graphic below.) Measure “position” along track for each of these height measurements. Create a simple table to calculate relative height and length change to find three different \( H/L \) calculations. Average them. Record your measurements, average, and standard deviation on this workbook page.

Also use this workbook page with a text box to write out your calculation for \( U \) (again in mJ.) Record your thoughts and necessary measurements.

Set \( U_{o} \) to be where cart is lowest on track. (What is position there read by motion sensor?)

\[ U(x) = ? \] where \( x \) is measured by Motion Sensor.

Program Calculator for \( U \). Be sure to click “New” calculation so as not to overwrite KE Sensor you created earlier!

**Figure 2: Calculate \( \sin \theta \)**

\[
\begin{align*}
\sin \theta &= \frac{(h_1 - h_0)}{(x_1 - x_0)} \\
\sin \theta &= \frac{(h_2 - h_1)}{(x_2 - x_1)} \\
\sin \theta &= \frac{(h_2 - h_0)}{(x_2 - x_0)}
\end{align*}
\]

**WORKBOOK PAGE 4: Plot U, KE, and \( E_{total} \)**

Before plotting all three, two things need to be done.

1. Create an \( E_{total} \) Calculation “Sensor.”
2. Make sure Properties for KE, U, and \( E_{total} \) all match (except their names, of course.) Be careful that they each have units for Y value being mJ and X value based on Time measured in s. Watch movie again, if needed.

Create a plot of U, KE, and \( E_{total} \) so that all three are superimposed on same graph. If all three are not automatically superimposed on same axes, then you probably need to check Properties for each Calculation so that units match. Both mJ and s need to be present for each Energy Calculation.

You are about ready to perform a run where cart starts at bottom of track and is sent up the incline where it comes to rest and then naturally goes back down incline.

Take care to consider start/stop conditions so as to have a clean data set. We recommend Start and Stop conditions as follows:

Start = Falls Below 1.7 m
and Stop = Rises above 1.7 m.

(Remember, “Rises Above” is relative to motion sensor, not height of cart along incline...) To get to Start and Stop conditions, simply double-click on StopWatch display. Find “Delayed Start...” and “Automatic Stop...”

Be careful to catch cart! Please practice with a gentle push and slowly increase force so as to get cart high up incline, but do not crash cart into sensor, please. After pushing, release cart before software
collects data; catch cart when it passes similar point of data collection on cart’s way back down. If you have set Start/Stop conditions properly, data should appear very clean and almost parabolic (at least for U and KE. What should Etot look like?)

WORKBOOK PAGE 5: Discuss Plot U, KE, and Etotal
Discussion should be 300+ words.
Add a text box (use workbook toolbar on right of workbook window) to discuss the three energies plotted. Be qualitative (conceptual) and quantitative (measurements.)

WORKBOOK PAGE 6: Discuss KE
Show a plot of just KE. Use “Data” button in Graph Toolbar to bring in data run to KE plot you create. You will investigate Kinetic Energy for one complete cycle, up and down track. Do your best to start and stop your analysis at same position point along track. (You may need to plot Position data also to help in this.)
Add a text box below plot.
Find loss in Energy due to friction. Show this as measurement as well as a Percent Loss; where percent loss is ratio of (loss to initial) * 100%.
Calculate work done by friction. Show your work in same text-box.

WORKBOOK PAGE 7: Discuss KE and U
Show a plot of U and KE. You will investigate Energy exchange for each half of one complete cycle (up and down track.) Do your best to start and stop your analysis at same position point along track. (You may use Potential Energy data to help in this.)
Use text box below plot.
Find loss in Energy due to friction in two situations:
1) On way up the track
2) On way back down the track
Show each of these as measurement as well as a Percent Loss; where percent loss is ratio of (loss to initial) * 100%. (Where initial energy for going up is energy at bottom of track; and initial energy for trip back down is energy cart had at top of track.)
Find work done by friction going up the track versus down the track. Show your work in same text-box.
Describe qualitatively and quantitatively Work done by Force of Gravity on way up track and then once again for way back down track.
Describe differences (again both qualitative and quantitative) between force of gravity and force of friction.

WORKBOOK PAGE 8: Finale- Conservation of Energy
Consider freebody diagram you drew back on WorkBook Page 2. Create a single plot of a single variable versus time (Use X, V, U, KE, or Etot; but only use one.) to illustrate concept of conservation of mechanical energy as well as that of non-conservative forces. Hopefully you will observe from plot an interesting shape that can be explained by freebody diagram. Create a lengthy discussion of these topics below plot.
Energy

Lab Station Number:

Names Sorted Alphabetically (by last names)

<table>
<thead>
<tr>
<th>First Name</th>
<th>Last Name</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
</tbody>
</table>

Date Data Was Taken:

Instructor's Name:

Lab Time:

Pages are stapled in upper left corner and are in this order (5pts):
Free Body Diagram (Include Difference in two Net Forces along inclined plane.)
Mass, KE Equation, U Equation and Sin(Theta) data table.
Plot of U, KE, and Etotal
Discussion of U, KE, and Etotal
KE (with answers to questions)
KE and U (with answers to questions)
Finale

Properly Finish This CoverSheet (5pts)
The Analytical Balance

Purpose
Find static balance for two types of systems: one dimensional and two dimensional. Balance will exist when forces (translational) and torques (rotational) are in equilibrium.

Equipment
- Meter stick
- Two 50[g] mass hangers
- Mass set similar to this: 1x 1g; 2x 2g; 1x 5g; 3x 10g; 3x 20g; 5x 50g; 7x 100g
- Digital or analytical mass balance (scale)
- One meter stick hanger (to suspend meter stick and spring balance)
- Spring Balance (spring mass scale)
- Odd shaped board
- Circular Bubble Level
- Pole Stand
- Long Pole and Short Pole
- Right Angle clamp
- Three small loops of string (about 8cm in diameter)

Theory
Most people have used an analytical balance, whether they know it or not. The old-style balance used to measure food is one example of an analytical balance.

We might think of laws and judges upholding a “just” balance. (Or, so we hope!) We expect in these situations pictured above that the forces are balanced. What do we really mean when we say “the forces are balanced?”

If a system is “balanced,” we are really saying that both translation forces and torques (the rotational analogue to forces) are in equilibrium. Weight is an example of a translational force. The Weight applied at a Distance from some pivot point is an example of torque. Many words are necessary to describe a torque compared to a force.
Force Example: $W = mg$
(Where both $W$ and $g$ are vectors; each contains both magnitude and direction.)
**Torque** is also a vector. Torque is given the symbol $\tau$ (tau).
Torque is the cross product of a momentum arm with its force.

$$\tau = \mathbf{r} \times \mathbf{F}$$

We say this as “torque is $r$ cross $F.” The cross product of two vectors yields a vector.
Force and Moment Arm are both vectors. $r \times F$ yields torque which is also a vector.

Another type of vector multiplication is the dot product which yields a scalar quantity. Work $[J] = \mathbf{F} \cdot \mathbf{d} = F \ d \cos(\theta)$

The magnitude of torque is the product of the magnitudes of both $\mathbf{r}$ and $\mathbf{F}$ and the sine of the angle between $\mathbf{r}$ and $\mathbf{F}$.

$$|\tau| = |\mathbf{r}||\mathbf{F}|\sin(\theta)$$

The direction of torque is in a third dimension orthogonal to the plane created by $\mathbf{r}$ and $\mathbf{F}$. We cannot change the order of a cross product. $\mathbf{r} \times \mathbf{F}$ is correct. $\mathbf{F} \times \mathbf{r}$ is not the same thing and is not torque. Only the order $\mathbf{r} \times \mathbf{F}$ will point the torque vector in the correct direction. (There will be more on this later.)

Notice that the units of the cross product and dot product (length and a force measurement) yield $[\text{N m}]$ (Newton Meter). The difference: torque is provided with direction and we normally leave the unit as Nm, whereas Work is only a scalar value and we often change Nm to be $J$ (Joule) for Energy.

Again, whether we know or it not, we have been using torques ever since we were each small children. At some point we probably learned that when using a wrench to turn a bolt, in order to maximize the torque (Or a hammer and nail, or opening a door, or a sea-saw...) we wanted to place our force far away from the bolt and apply our force at right angles to the wrench. Which hand has a force that will maximize the torque on the bolt?

Hand B is farthest from the bolt. We can say that hand B has a longer *moment arm* (also called a *lever arm*). Hand B will provide the larger torque on the bolt.

The Moment Arm is the vector drawn from the *fulcrum* to the *force*. (Be careful! DO, “draw arrow from datum towards the force.” Do NOT “draw arrow from force to pivot point.”)
*pivot point* and *fulcrum* mean the same thing in this experiment. The term *datum* is also used. Datum and fulcrum bring in the idea that our “pivot point” can be placed anywhere mathematically. You will use the concept of the versatile datum in problems two and three of this experiment.

As the moment arm increases in length, the torque also increases.

In our equations \( \tau = \vec{r} \times \vec{F} \) and \( |\vec{r}| = |\vec{r}| |\vec{F}| \sin(\theta) \) we see that the angle between \( \vec{r} \) and \( \vec{F} \) is also important. Maybe you learned at a young age that one of these two hands seen below will apply more torque. Which picture applies more torque, assuming both forces have the same magnitude?

When force is applied at right angles to the moment arm, torque is maximized. This can also be seen mathematically:

\[
|\vec{r}| = |\vec{r}| |\vec{F}| \sin(\theta)
\]

\[
\sin(90^\circ) = 1 \\
\sin(30^\circ) = 0.5
\]

Our goal in today’s experiment is to make certain that each force is set to be perpendicular to the moment arm. This will not only maximize the torque, it will also make calculation and measurement easier since we won’t have to measure the angle between force and moment arm and because \( \sin(90)=1 \) could not be easier.

If the angle between \( \vec{r} \) and \( \vec{F} \) is 90°, then we may use

\( \tau = rF \)

The direction of torque is in a third dimension orthogonal to the plane created by \( \vec{r} \) and \( \vec{F} \). We use the “right hand rule” to discern the direction of torque. The right hand
rule says that we can point our four fingers in the direction of \( \mathbf{r} \) and then curl our fingers in the direction of \( \mathbf{F} \). The thumb sticking out will then point in the direction of \( \mathbf{\tau} \). Torque is in the third dimension. When \( \mathbf{r} \) and \( \mathbf{F} \) are pointing as pictured below, torque is inward, towards the paper. We call this a negative torque causing a clockwise rotation of our system.

Try this with your right hand. Point your four fingers in the direction of \( \mathbf{r} \), then curl them down in the direction of \( \mathbf{F} \), your thumb will point into the paper. We denote this torque's vector direction with an “x” symbol. We can also show a clockwise rotation arrow about the datum. We can also say the torque going into the paper is negative.

This is a negative torque creating a clockwise rotation and denoted with an X.
Think of the X as the feathers of an arrow going away from you INTO the page.
A positive torque will be represented with a dot or counter-clockwise arrow. (The dot represents the tip of an arrow head; this is the view you might prefer NOT to see with a real arrow!) Positive torque creates a counter-clockwise rotation and the vector direction of torque is coming out of the paper towards you. Try the right-hand rule in the picture below. Your thumb should be pointing away from the paper when you are finished with crossing \( \mathbf{r} \) into \( \mathbf{F} \).

\[
\text{This is a positive torque}
\]
\[
\text{creating a counter-clockwise}
\]
\[
\text{rotation and denoted}
\]
\[
\text{with a dot} \cdot.
\]
\[
\text{Think of the} \cdot \text{ as the}
\]
\[
\text{tip of an arrow coming}
\]
\[
\text{straight towards you}
\]
\[
\text{OUT of the page.}
\]

If a system is in equilibrium, two statements must hold true:

1) Translational (linear) Forces must sum to be zero.
\[
\sum \mathbf{F} = 0
\]

2) Torques must sum to be zero.
\[
\sum \tau = 0
\]

For each of the problems you do with torques, decide whether vertically up or vertically down is positive. Also, decide on the datum*. Remember, the datum is the mathematical point from which you measure all moment arms. And remember that a counter-clockwise torque about the datum is positive, while a clockwise torque about the datum is negative.

Problems one and two will deal with a meter stick, a one dimensional system.
Problem three will deal with a two dimensional, non-uniform object.

*Oh! We will choose the datum for you. On problem one, the datum is also the pivot point for our system. The fulcrum in problem one will be the location of the vertical hanger. In problem two we will move to the datum to be the 0 point on the meter stick. Problem two will force you to use both equilibrium conditions in tandem (simultaneous linear equations): the sum of the forces must equal zero and the sum of the torques must equal zero. On problem three, you get to set the datum at the origin of your graph.
**THEORY** Problem One: What is mass of the meter stick?

L is the lift force required for equilibrium.

\[
A = ? \text{[gram]}
\]

\[
a = (\text{about}) 10 \text{cm}
\]

\[
b = ? \text{[cm]}
\]

\[
B = ? \text{[g]}
\]

Find A and a and b experimentally. Don’t forget the mass hanger itself has mass (50[g]).

B represents the weight of the stick. Be careful: B is not necessarily at 50cm because B acts at the true center of gravity for the stick.

In solving for problem one, sum the torques starting from the left moving to the right. The sum of the torques should be zero. Do all your math for problem one using symbols, (a, A, b, B) instead of numbers. Solve completely for B, then write down that generalized equation. Evaluate B by using the values for a, A, b. Then measure B on the scale. Compare the two B’s with a percent difference. Box all the important results.
THEORY Problem Two: The Analytical Balance

In problem two you will place the mathematical datum at the zero point for the meter stick. All moment arms will be measured from there. This will force you to use both the sum of the torques and the sum of the forces because \( L \) will have a torque at \( 30 \, \text{cm} \), but \( L \) will not be known. You must then mathematically find \( C \) at \( 90 \, \text{cm} \). Again, start at the far left and sum the torques and forces as you move to the right. Do all your math for problem two using symbols (\( a, A, l, L, b, B, c, C \)) instead of numbers. Solve completely for \( C \), then write down that generalized equation. Finally, evaluate \( C \) by using the values for \( a, A, l, L, b, B, c \).
THEORY Problem Three: Center of Gravity (C.G.) for a Two Dimensional, Irregular Object

The theory behind problem three uses the sum of the torques; but we evaluate the torques in each dimensions separately.

Let’s take an irregular shaped object and consider where the center of gravity is in just two dimensions.

To find this 2-D C.G., we would start by measuring the weight at three points. Which points we use doesn’t really matter; but in the scooter picture above, the wheels are most convenient. (We could use four points, if this were a four-wheeled scooter.) Next we create a Cartesian Coordinate Graph to show where those three scales exist relative to one another. We then arbitrarily choose an origin on the graph and show the X and Y axes.

We can find the torque that each wheel creates about the X axis. Similarly, we find the torque that each wheel creates about the Y axis. Remember that torque is
simply $r \cdot W$; and keep in mind some torques will be negative and others positive. (We would want to be certain the force pushing down on the scales was indeed vertical (or plumb), thus providing a right angle so that we can ignore the angle when evaluating torque.)

Finally, we employ the torque concept again to find the X coordinate for C.G.:

$$X_{C.G.} \cdot \text{weight of object} = \text{Net Torque object has about the Y-axis}$$

The $X_{C.G.}$ is simply the X-Coordinate for the Center of Gravity. The Net Torque the object has about the Y-axis is simply the sum of the torques found by each wheel’s weight multiplied by each wheel’s moment arm (x-displacement) as measured along the x-axis from the origin.

$$\text{Net Torque about the Y-axis} = w_1x_1 + w_2x_2 + w_3x_3 = \sum_{i=1}^{n} w_i x_i$$

The weight of the object is simply $w_1 + w_2 + w_3 = \sum_{i=1}^{n} w_i$

Again, we know:

$$X_{C.G.} \cdot \text{weight of object} = \text{Net Torque object has about the Y-axis}$$

Solving for $X_{C.G.}$ yields:

$$X_{C.G.} = \frac{\text{Net Torque about the Y-axis}}{\text{weight of object}} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

$Y_{C.G.}$ yields:

$$Y_{C.G.} = \frac{\text{Net Torque about the X-axis}}{\text{weight of object}} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}$$

See the graphic below for the scooter problem worked out fully.
\[
(\frac{(8) \times 50 + (-2) \times 92 + (5) \times 87}{50 + 92 + 87}) = 2.8 = X_{C.G.}
\]
\[
(\frac{(2) \times 50 + (-3) \times 92 + (-9) \times 87}{50 + 92 + 87}) = -4.2 = Y_{C.G.}
\]
**PROCEDURE:**

**PROCEDURE Problem One: What is mass of the meter stick?**
Use the provided cover sheet. You will have five pages, including the cover sheet, when this lab WriteUp is complete.

Problem One will use one entire sheet of paper.

Draw a picture similar to the one in the theory section for problem one.

Find the Center of Gravity (C.G.) for the meter stick. You may employ one or both of the methods mentioned below.

**C.G. Method One:** Hold the meter stick horizontally in front of you, with both hands such that one hand is at each end of the stick. Point the forefinger on each hand and support the stick on your forefingers. Slowly bring your fingers together. The C.G., torques, normal forces, static friction and kinetic friction all interact so that when your fingers meet, they will meet at the C.G. Try it quickly too.

**C.G. Method Two:** Mount the stick in the metal holder and hang the holder from the horizontal pole. Slide the stick in the metal holder until you find the balance point.

1) Write down the stick’s C.G. directly on your graphic for problem one.
2) Slide the support hanger to be at 20 [cm] mark.
3) Use the hanger at 20[cm] to hang the stick from the horizontal pole. Gently tighten the thumb screw, but don’t make it too tight.
4) Hang a 50 [g] mass hanger at about 10 [cm]. We will call this mass A.
5) Now add mass to mass A until the stick is static and horizontal when you let go. You may need to carefully slide mass A a hair’s breadth this way or that way to accomplish a stable, horizontal system. Nobody breathe! Turn off the A.C.! Or... simply don’t let anyone bump your table.
6) Record the total Mass A value in grams on your data sheet for problem one. (Don't forget the mass hanger itself is about 50 [g].)
7) Record the position of Mass A. You will use this to find the vector \( \mathbf{a} \).
8) On your paper for problem one, sum the torques starting from the left moving to the right.
9) Set the sum of the torques equal to zero.
   Do all your math for problem one using symbols \((a, A, b, B)\) instead of numbers. Solve completely for B, then write down that generalized equation for B as a function of \((a, A, b)\) Box this function.
10) Evaluate \( B_{\text{analytical}} \) by using the values for \( a, A, b \).
11) Then measure \( B_{\text{scale}} \) using the provided scale. Should you remove the hanger before measuring \( B_{\text{scale}} \)?
12) Compare the two B’s with a percent difference.
   \[
   \text{Percent Difference in } B = \frac{|B_{\text{scale}} - B_{\text{analytical}}|}{B_{\text{avg}}} \times 100\%
   \]
13) Box all the important results.
14) Answer this question on your sheet for problem one: Which value for B do you trust more? You need to decide, because that is the value of B you need to use for problem two. You will also use, in problem two, the C.G. position you found in problem one.

15) Answer these questions on your sheet for problem one: What would change within the mathematics if your meter stick was static (not moving) but the stick was not level? What new measurement problems are present?

16) On your sheet for problem one: quantify the measurement uncertainty in \( B_{\text{analytical}} \). Compare this uncertainty to the percent difference.

**PROCEDURE Problem Two: The Analytical Balance**

Draw a diagram for problem two similar to the diagram in the theory section for problem two. Label all the vectors using a, A, l, L, b, B, c, C.

Do NOT set up the meter stick and masses until you have finished all the math below and have found C in grams.

In problem two you will place the mathematical datum at the zero point for the meter stick.

Draw the datum.

All moment arms will be measured from there. This will force you to use both the sum of the torques and the sum of the forces because L will have a torque at 30[cm], but L will not be known. You must then mathematically find C at 90[cm].

After your drawing is fully labeled, sum the torques and sum the forces. Again, start at the far left and sum the torques and forces as you move to the right.

Do all your math for problem two using symbols (a, A, l, L, b, B, c, C) instead of numbers. Solve completely for C, then write down that generalized, model equation for C. Be sure to simplify your final equation.

Finally, use your model equation for C to evaluate C using the values for a, A, l, b, B, c. Use the values given in the theory section for a, A, l, c. Use the values from problem one for b and B.

Box the symbolic equation for C and the final results.

Do all this work neatly on your sheet for problem two.

Now, set up the system to see if your value for C works. Is your system level and static? If it didn’t work, then back to the drawing board...

Answer these questions on your sheet:

How close was your value for C to making a balanced, level system?

How far did you have to move A, L, or C to get a balanced, level system?
How does the generalized equation benefit you, if you wanted to change values for $a$, $A$, $l$, $c$?

How does simplifying your model equation for $C$ help improve the accuracy of your answer? (Think in terms of measurement uncertainties.)

Quantify the measurement uncertainty in $C$.

**PROCEDURE Problem Three: Center of Gravity (C.G.) for a Two Dimensional, Irregular Object**

Since we could not afford to purchase for everyone a three-wheeled scooter, we will use a very expensive board that will be left on the lab table when you are finished. This board is so expensive because it has an irregular shape. Because of this expense we ask two things: (1) Do NOT write on the board. (2) Do NOT remove the screw hooks for any reason.

Start by finding the mass of the board. Place the board onto the digital scale and record the result on your sheet for problem three. You might consider whether or not you will put the bubble level on the board when you find the mass of the board.

Next, you will place a piece of graph paper onto the board. You will poke the hooks right through the paper and thread the strings through the holes. But first! Let’s consider strategic placement of the graph paper.

After studying these two equations, what things can you employ while positioning your graph paper onto the board in order to reduce the error propagation within your calculations for $X_{C.G.}$ and $Y_{C.G.}$?

$$X_{C.G.} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

$$Y_{C.G.} = \frac{\sum_{i=1}^{n} w_i y_i}{\sum_{i=1}^{n} w_i}$$

(Think about this for a couple of minutes and debate amongst yourselves, then watch the video on the Analytical Balance web page for increasing accuracy by reducing error propagation.)

With your newfound knowledge, and WITHOUT removing any screw hooks, place a piece of graph paper onto the board. You will poke the hooks right through the paper and thread the strings through the holes.

Next, choose an origin and draw your $X,Y$ coordinate system. (Again, the strategy—a great new word—you apply here might increase accuracy by reducing error propagation.)
Use the grid lines on the graph paper to measure the X and Y coordinates. Write these coordinates directly on the graph paper. Your graph paper will stay on the board until after the conclusion of this experiment. Do NOT remove the paper until after you are finished with the experiment. You will then take off the paper and include it with your WriteUp.

Next is hardest part of the day. Carefully measure values for w₁, w₂, and w₃, the weight (or mass) at each hook. Follow these guidelines:

- Hang the spring balance from the horizontal pole and slide the right angle piece down a bit until the spring hook and the board hanging from the hook will be just a little above the table.
- Hang the board onto the spring balance hook using one of the strings/hooks on the board.
- Hold the other two strings with your hand.
- Do NOT touch your hands to the board. Let nothing touch the board (save the strings, graph paper, and bubble level.
- Hold the strings perpendicular to the board.
- Place the bubble level on the center of the board.
- When the spring balance is plumb* (NOT EASY TO DISCERN!), and the bubble level is level, take a reading on the spring balance. (I prefer the gram side to the Newton side because calculations are a lot easier.)
  * A plumb spring balance (orthogonal to the board) can only be discerned by looking at it from multiple sides to see if the board needs to be moved right, left, forward, or backward to get the spring balance plumb.

- Compare the sum of w₁, w₂, and w₃ with the digital scale value you got for the mass of the board. Your value should be within 5% or less. Record these two values and their percent discrepancy on the sheet for problem three. If your discrepancy is large, you need to repeat the weight measurements at each hook.

Record these weights on the board next to each hook. (I mean, on the graph paper!)

Calculate the Xₐ.G. and Yₐ.G. for your system. Record these values on your graph paper as the C.G. coordinates.

Calculate the uncertainty in Xₐ.G. and Yₐ.G. (Use error propagation. Find the uncertainty in each length and each weight measurement that went into finding the X and Y C.G.’s. Add the relative uncertainties.)

Draw a big box centered on your C.G.. Make the size of the box appropriate for your uncertainties.

Turn the board upside down and see if the board will balance on the vertical pole somewhere within your C.G. box.

Record the answers to these questions on your sheet for problem three.
(Some students do all their work on the graph paper. Others have two sheets for problem three: a worksheet similar to problem one and two and also a graph paper with holes, coordinates, weights, CG box, et. al.)

Did your system balance at the C.G.? If not, what was the worst offending issue?

What method above was used to check the accuracy of your values for $w_1$, $w_2$, and $w_3$? (This might be a PostLab question, too!)

How important do you think the C.G. is for an Airplane? Small Airplanes? Large Airplanes? Why?
See the web page for this experiment for some links.

If you try to balance a broom on your finger, what role might the C.G. of the broom have in relation to your finger?
Analytical Balance

Lab Station Number:

Names Sorted Alphabetically (by last names)

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Date Data Was Taken:

Instructor's Name:

Lab Time:

elements: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):

Problems 1-3 and supporting graph paper for #3 (with holes and annotations.)
Analytical Balance Problem 1:
View:ToolBars:Drawing
Use tools to draw pictures, vector arrows, text boxes, etc.

![Analytical Balance Diagram](image-url)
Analytical Balance Problem 2:
View:ToolBars:Drawing
    Use tools to draw pictures, vector arrows, text boxes, etc.
Analytical Balance Problem 3:
View: ToolBars: Drawing
Use tools to draw pictures, vector arrows, text boxes, etc.
Rotational Inertia

MUSCD Physics

April 9, 2004

1 PURPOSE

The rotational inertia of a metal disk will be measured in three different methods. The first is by Newton's Second Law for Rotation. The second is by direct measurement of mass and shape. The third is a refinement of the shape measurement.

2 THEORY

The rotational inertia (or moment of inertia in some textbooks) of a rotating object is the rotational equivalent of mass. When a force $F$ acts on a mass $m$, it causes an acceleration $a$, as obtained from Newton's Second Law for translational (or linear) motion,

$$a = \frac{F}{m}$$

In the rotational case, a torque, $\tau$, is applied to an object of rotational inertia, $I$, resulting in an angular acceleration, $\alpha$, given by,

$$\alpha = \frac{\tau}{I}$$

2.1 Finding $I_1$

In this experiment, a falling mass will be used to accelerate a large disk that has small, dense hubs. The acceleration, $a$, of the falling mass, $m$, will be measured using a Smart Pulley. The mass will accelerate due to the sum of the forces acting on the system. There is the weight of the mass $m$, the tension in the string $T$, and friction in the bearings of the large wheel $f$. We will deal with friction later.

$$ma = mg - T$$
$$T = m(g - a)$$

The torque $\tau$ acting on the disk is

$$\tau = TR = m(g - a)R; \quad (1)$$

where $m$ is the falling mass; and $R$ is the disk’s Radius.
The angular acceleration $\alpha$ of the disk is related to linear acceleration $a$ by the equation that links translational motion to rotational motion

$$\alpha = \frac{a \, \text{rad}}{R \, \text{s}^2}.$$  

Newton’s Second Law for rotation can be solved for the torque,

$$\tau = I\alpha$$  

(2)

When written this way, it has the form of the equation of a straight line

$$y = mx + b,$$

When $\tau$ vs. $\alpha$ is plotted, the slope of the line is equal to the rotational inertia, $I$.

But what about friction? Friction applies a torque too. The equation is now:

$$\tau_{\text{driving}} - \tau_{\text{friction}} = I\alpha$$

$$\tau = I\alpha + \tau_{\text{friction}}$$

You will vary $m$ and calculate the resulting $\alpha$’s from your measurements. You will plot $\tau$ versus $\alpha$. The slope should be $I$. What should the vertical intercept be?

$$\tau = I_1\alpha + \tau_{\text{friction}}$$  

(3)

where

$$\alpha = \frac{a}{R}$$  

(4)

$$\tau = m(g - a)R.$$  

(5)

2.2 Finding $I_2$

The second method uses direct measurement. A disk or cylinder has a theoretical rotational inertia of

$$I_2 = \frac{1}{2}MR^2.$$  

(6)

The mass and radius of the wheel can be directly measured. The rotational inertia $I_2$ can be calculated and compared with the slope of the line from the first method.

2.3 Finding $I_3$

$I_1$ is to be found via measurements, calculations, and behaviour (as seen in the plot of $\tau$ vs. $\alpha$.) $I_2$ is to be found via measurement and calculation. But $I_2$ can be considered an overestimate, why? The equation $I_2 = \frac{1}{2}MR^2$ assumes that all the mass of the disk is uniformly distributed throughout the large radius $R$. But, in fact, there are small yet massive hubs permanently attached to the wheel. $\frac{1}{2}MR^2$ takes this smaller radius $R$ mass $m$ and distributes it throughout $R$. Thus $I_2$ is an overestimate.
Let’s build a corrected $I_2$ and call it $I_3$. It will be the rotational inertia of the big wheel plus the rotational inertias of the small hubs. We will need to build our equation in terms of as many “knows” as we can.

Note: We will assume that the small hubs have uniform density (which is not the case.) We don’t know the Mass of the big $R$ portion, but we do know the mass of the total wheel $M_T$.

$$M_R = M_T - 2m_r$$

Let’s sum the rotational inertia of the big wheel (with Mass=$M_T - 2m_r$) and the rotational inertias of the small hubs with mass $m_r * 2$.

$$I_3 = I_R + 2I_r$$

$$= \frac{1}{2} M_R R^2 + 2(\frac{1}{2} m_r r^2)$$

$$= \frac{1}{2} (M_T - 2m_r) R^2 + m_r r^2$$

$$= \frac{1}{2} M_T R^2 - m_r R^2 + m_r r^2$$

$$= \frac{1}{2} M_T R^2 - m_r (R^2 - r^2)$$

But our $I_2$ was really $\frac{1}{2} M_T R^2$, so

$$I_3 = I_2 - m_r (R^2 - r^2).$$

(7)

We can measure/calculate $I_2$, $R$, and $r$. But how do we find $m_r$? Instead of taking a hack saw and cutting off the hubs to measure their mass, let’s approximate their mass using density. But, we don’t know the density of the metal, so we will have to “use the back door.”

$$\text{Density} = \rho = \frac{\text{Mass}}{\text{unit Volume}}$$

$$\text{Volume} = \pi * \text{Radius}^2 * \text{Width of Cylinder}$$

$$V_R = \pi R^2 H$$

$$V_r = \pi r^2 h$$

where $H$ is the width of the big wheel and $h$ is the width of the hub.

Now that we have some base equations for volume and density, let’s find $m_r$.

Because $\rho = \frac{\text{Mass}}{\text{Volume}}$,

$$M_T = 2m_r + \rho V_R.$$
Then, Solving for $m_r$,

$$m_r = \frac{M_T - \rho V_R}{2}$$

$$= \frac{M_T - \frac{M_r}{V_{R+2V_r}} V_R}{2}$$

$$= \frac{1}{2} M_T (1 - \frac{V_R}{V_R + 2V_r})$$

$$= \frac{1}{2} M_T \left( \frac{V_R + 2V_r - V_R}{V_R + 2V_r} \right)$$

$$m_r = M_T \left( \frac{V_r}{V_R + 2V_r} \right)$$

Finally, After substituting in all the Volumes (where all the $\pi's$ cancel), we are left with

$$m_r = M_T \frac{r^2 h}{R^2 h + 2r^2 h}$$

### 3 EQUIPMENT NEEDED

- Rotational Inertia Apparatus
- Macintosh computer, SW750 Interface, Capstone Software, Smart Pulley
- Hanger, 100g mass, and 8 washers ($\approx 5g$ each)
- Physics string
- Digital Mass Balance
- Vernier caliper
- Large vernier caliper (One for whole room; don’t move it!)

### 4 PROCEDURE

#### 4.1 Setup

- Turn on the interface. Open the document titled “Rotational_Inertia.cap”. Make sure Data Studio is not open!
1. Plug in the smart pulley into digital channel 1.

2. Position the rotational inertia apparatus and the smart pulley so that when the string is wrapped around the large disk, it passes over the smart pulley and attaches to the mass hanger. Be sure the weight can fall freely, and that the string is long enough to accelerate the disk for many turns. But make sure it will not allow the hanger to hit the floor.

3. Carefully loosen the large, hand-turned knob so as to release the large wheel.

4. Measure the mass of the large wheel using the digital balance at your lab station.

5. Measure the diameter of the large disk using the large vernier caliper at the front desk (Take your disk to the front, do not move the large vernier calipers away from the front desk).

6. Back at your table, measure the other dimensions of the disk: width (thickness) of large disk, diameter of small hub, and width of small hub.

7. Carefully replace the large disk; and gently tighten the hand–turned knob.

8. Have your instructor verify proper axle placement before taking data!

9. Use the brake on the rotational inertia apparatus or your finger to hold the disk stationary. The brake is spring loaded. You rotate it about a quarter turn and the spring pushes it into the disk. Gently pull it out and rotate to secure away from the disk.

10. Put 100 g on the mass hanger. Measure their mass together; and record in the data table for trial one (in kilograms!)

4.2 Data Recording

1. The file “Rotational_Inertia.cap” will already have a graph of Velocity versus Time created for you.

2. With the disk stationary, click the “Record” button; then release the disk. Data collection will commence when the pulley starts rotating and will cease when the rotation of the pulley changes direction.

3. Scale-to-Fit the graph by clicking the “Scale-to-Fit” button.

4. There will be one data point created when the pulley rotation changes direction, we do not need this data point. We will use the “Highlight” button to highlight only relevant data.
(a) Click the “Highlight” button. A transparent box will appear on the graph. Note: The more you click the “Highlight” button the more boxes will appear, you will only need one!

(b) Move the transparent box over the the data that is relevant, in this case it is only data that is linear, you can change the size of the transparent box by moving to one of its corners and dragging it in or out to an appropriate size.

(c) Delete, you can delete the highlight box or any other object you no longer need from your graph by clicking on the object in question and using the “Forward Delete” key on your keyboard or the “Delete Object” button.

5. Once you have highlighted the data you are ready to find the slope of your data using the “Line Fit” tool. The line fit button has two parts:

(a) An activate part, the area circled, activates the line fit function.

(b) A drop down arrow, the area circled, will display several options to fit data, we will be using the “Linear: mx+b” option.

6. Rename the data from “Run#1” to be the mass value in kilograms by double clicking on “Run#1” in the Data Summary area left of your graph. This will make it easier to analyze the data.

7. Only for trial one:

   This step is for archiving a sample of trial one. Don’t do this for any other trial!

   Annotate the graph by explaining what it represents by using the “Annotate” button. Click anywhereon your graph to make it active and copy it to the clipboard by clicking the “Copy” button located at the very top center of the Capstone work page. In the Excel spreadsheet, go to the tab “Acceleration Example”, and press command+v.

8. Clear the data from the graph. Don’t delete the data! Just clear it using the graph’s “Data” button. A drop down of previous runs will appear if you click on the drop down arrow, you can uncheck runs that you do not wish to display.
9. Delete ALL runs that are bad. Keep only the valid runs. You don’t want to collect more than nine runs total! You can delete runs by using the “Delete Last Run” button, or use the arrow to open a drop down menu to pick and choose other runs. If you accidentally delete a run you intended on keeping, just use Command+z (Mac) or Ctrl+z (PC) to undo.

10. Add a washer (≈ 5g) to the mass hanger and re-mass the hanger system.

11. Repeat the experiment. Continue adding one 5g washer and repeating experiment until you have at least nine trials. Do not exceed 200g for the mass hanger!

5 ANALYSIS

1. Use data collected and equations from theory section to fill in the table.

2. Make a plot of $\tau$ vs. $\alpha$. Determine the slope of the line with a linear trend fit. This should be equal to the rotational inertia, $I_1$. Include a physics translation.

3. Calculate $I_2$.

4. Add to the plot the data of $\tau_3$ (This is the data from $\tau = I_2\alpha$.) Do not add a trendline; this should be linear already! Instead, reformat the data to appear as a line instead of data points. Include a physics equation. (You already know the slope and vertical intercept.) You can add this data to the plot by choosing “Source Data” from the “Chart” menu. The “Chart” menu appears when the plot worksheet is active.

5. Compare the two values, $I_1$ and $I_2$.

6. Calculate $I_3$ using the theory provided to you on the web or in this manual.
# Rotational Inertia

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**Names Sorted Alphabetically (by last names)**

**Date Data Was Taken:**

**Instructor's Name:**

**Lab Time:**

examples: T8, W4, R10

**Pages are stapled in upper left corner and are in this order (5pts):**

- Data Table
- Annotated Sample Picture from DataStudio
- Plot of Torques versus Angular Acceleration
  
(No other pages!)

Properly Finish This CoverSheet (5pts)
### Constants:

- Mass of Wheel (kg) =
- Radius of Wheel (m) =
- Width (thickness) (m) =
- Radius of hub (m) =
- Width of hub (m) =

### Table

<table>
<thead>
<tr>
<th>Column Name</th>
<th>Explanation or Formula</th>
<th>Example Trial One</th>
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<td>Acceleration (m/s²)</td>
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<td>Comparative τ² (Nm)</td>
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### Uncertainty in Acceleration: Trial 9 using 8 attempts

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<th>Average Acceleration Trial 9 with 8 attempts:</th>
<th>Uncertainty in Accel.</th>
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### Plot τ vs. α and report results below:

- Slope = #DIV/0!
- Intercept = #DIV/0!

### The equation that describes this plot of my data is:

- Rotational Inertia = \( I_1 = \) = slope of
- Rotational Inertia = \( I_2 = \) =

% difference = 

The side portions of the disk were included in your measurement (calculation) of \( I_2 \). Write out an equation for the rotational inertia; only this time let it properly represent the side portions. Call the total mass of the wheel \( \text{MASS} \). Call the mass of just one side portion \( \text{mass} \). Call the big radius \( R \) and the little radius \( r \). Call this calculation \( I_3 \).

- Hub mass = \( m = \) =
- Rotational Inertia = \( I_3 = \) =

Just looking at the form of \( I_3 \), should it be greater or smaller than \( I_2 \)? (In other words, was your \( I_2 \) an overestimate or underestimate for the rotational inertia of the disk?) Also, would the new \( I_3 \) then be closer to \( I_1 \) than \( I_2 \) was?
\[ ma = mg - T \]
\[ T = m(g - a) \]
\[ \tau = TR = m(g - a)R \]
\[ \alpha = \frac{a}{R} \]
\[ V_R = \pi R^2 H \]
\[ V_r = \pi r^2 h \]

\[ M_{\text{Total}} = M_R + 2m_r \]
PENDULUM : "WEIGH" THE EARTH

PURPOSE

In this experiment, you will empirically find g, the acceleration due to gravity and then calculate Mass of Earth. To find g you will use a pendulum.

THEORY

The Period \((T)\) of a pendulum is the time it takes for the pendulum bob to make one complete cycle, that is; forward then backward, returning to its original position. As the Length \((L)\) of the pendulum string changes, so does Period \((T)\). \(T\) is therefore a function of \(L\). The relationship between \(T\) and \(L\) is as follows:

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

Squaring both sides leaves

\[
T^2 = 4\pi^2 \frac{L}{g}.
\]

In this experiment you will find Period for several different lengths of string. Then you can plot \(T^2\) versus \(L\). The slope of this line is

\[
\frac{4\pi^2}{g}.
\]

From slope value you can solve for value of \(g\).

Isaac Newton's Gravitational relationship requires that two masses are attracted to each other with a gravitational force. He found that force was directly proportional to product of the masses and inversely proportional to square of distance between them. Newton named the proportionality constant \(G\). Force equation for gravitation is as follows:

\[
F = \frac{GMm}{R^2} = mg.
\]

Taking \(M\) to be mass of Earth and \(m\) to be mass of pendulum bob this equation can be solved for mass of Earth. Note that mass of the pendulum bob is not a part of finding mass of Earth.

Use \(R = 6.38 \times 10^6\) m and \(G = 6.67 \times 10^{-11}\) Nm²kg⁻².

EQUIPMENT NEEDED

- Clamp Stand with Protective Cardboard underneath
- Long and short support rods
- Right angle clamp
- String
- Mass for a pendulum bob
- Two Meter Stick
- Finger Clamp
PROCEDURE

Setup: Make a pendulum
1. If it is not already done for you, set up clamp stand (with protective cardboard underneath), long support rod, right angle support, and then a short support rod with a finger clamp.

2. Fix a mass-bob to a string and pinch the other end of the string in the finger clamp. Please DO NOT wrap string around anything.

Data Recording:
1. Measure length $L$ of pendulum from fixed end of string (finger clamp) to some point above knot that holds mass-bob. (One should measure to center of mass for pendulum bob; but we want to have something to talk about in Analysis step 8 below.)

   Note: You will systematically, incorrectly measure length; so mark string or use knot itself to keep a consistent length-offset.

2. Create an initial amplitude of less than 15° between the top of the string and the vertical direction. (See picture below.)

3. Let the pendulum go. Allow pendulum to swing a number of cycles before starting timer.

4. Time a total of 50 complete swings. (More periods will yield more accurate results.)

5. Record length and total time in data table.

6. Repeat steps 1-5 with various lengths until you have measured total time for 10 different lengths.

Notes: Lengths should vary from 15 [cm] to 98 [cm]. Let one of your lengths be equal to $L$ that comes from $T = 1$ [s]. Find this $L$ using $T^2 = \frac{4\pi^2 L}{g}$.

Use $g = 9.8$ [m/s/s].
ANALYZING THE DATA

1. In your table calculate columns for $T$ (period) and $T^2$.

2. Plot points of $T^2$ vs. $L$. Then draw your best fit line.

3. Find slope of your line.

4. Find $g$ from your slope and compare it to accepted value of $g$. Record these on data table.

5. Use your $g$ and Newton’s Gravitational Force equation to find mass of the Earth. Compare this with accepted value for mass of Earth = $5.98 \times 10^{24}$ kg. Does this comparison yield same number as comparison in previous step?

6. Add theory line to graph with appropriate physics equation (use decimal value for slope.)

7. Add text box to your plot using your non-theory line equation to find appropriate length to create a clock. Test this length by counting cycles for 10 seconds. Does it work? Report your results in same text box.

8. Visually show x-intercept. Add a textbox to mathematically describe how you calculate x-intercept and what it’s value is. Correlate meaning of x-intercept to something in your measurements. State value for purposeful Length_offset which you measured. Compare that to calculated L_offset. Include these quantities in text box.

Note: View movie for how to manipulate spreadsheet to view x-intercept. Basically: forecast trendline backwards by 1 unit; changed x-axis scale to have a forced minimum of -0.1 [m] and y-axis to start at -0.3 [s$^2$].
## Pendulum

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### Date Data Was Taken:

### Instructor's Name:

### Lab Time:

Examples: T8, W4, R10

### Pages are stapled in upper left corner and are in this order (5pts):
- Data Table
- Plot of Period^2 vs. Length including:
- Data with Fit
- Theory Line
- X-Intercept with comparison
- Length for Clock calculation and comparison

Properly Finish This CoverSheet (5pts)
<table>
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<tr>
<th>Trial</th>
<th>Cycles</th>
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<th>Stopwatch Total Time (min: seconds)</th>
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<th>Period (s)</th>
<th>Period^2 (s^2)</th>
<th>Theoretical Period^2 (s^2)</th>
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After plotting Period^2 versus Length I found that:

\[
\text{Slope} = 4\pi^2 / g = \#DIV/0! \\
\text{Initial Period}^2 = \text{Vertical Intercept} = \#DIV/0!.
\]

\[
g_{\text{Measured}} = \text{m/s}^2 \quad \text{This value comes from } g = 4\pi^2 / \text{Slope}
\]

\[
g_{\text{Accepted}} = \text{m/s}^2
\]

Discrepancy between the gravities.

Earth Mass_{\text{measured}} = 
Earth Mass_{\text{accepted}} = 
Discrepancy between the masses.
What equation for \( x \) would solve this equation?

This equation for \( x \) will solve the above equation.

Furthermore: In order for the cosine argument to be valid, \((k/m)^{1/2}\) must be in radians per second! Therefore, it must be an angular frequency \( \omega \).

Let \( x = \cos \left( \sqrt{\frac{k}{m}} \right) t \)

\( \dot{x} = -\sqrt{\frac{k}{m}} \sin \left( \sqrt{\frac{k}{m}} \right) t \) This equation for \( x \) will solve the above equation.

\( \dot{x} = -\frac{k}{m} \cos \left( \sqrt{\frac{k}{m}} \right) t \) Furthermore: In order for the cosine argument to be valid, \((k/m)^{1/2}\) must be in radians per second! Therefore, it must be an angular frequency \( \omega \).

\( \omega = 2 \pi f \implies f = \frac{\omega}{2 \pi} \)

\( Period = T = \frac{1}{f} = \frac{2 \pi}{\omega} \)

\( T = 2 \pi \sqrt{\frac{m}{k}} \)

\( T = 2 \pi \sqrt{\frac{m}{k}} \); because, \( \omega = \sqrt{\frac{k}{m}} \).
So, what does $k$, the pendulum's "spring" constant, equal?

\[ F = -kx \]

\[ x = \text{arc length} = l\theta \quad \text{Where } l \text{ is the pendulum length}. \]

\[ F = -kl\theta \]

\[ F = -mg \sin \theta \]

\[ kl\theta = mg \sin \theta \]

\[ k = \frac{mg \sin \theta}{l\theta} ; \text{ but, as can be seen from the next page} \quad \frac{\sin \theta}{\theta} = 1. \]

\[ k = \frac{mg}{l} \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

Substituting in the value of $k$ above yields:

\[ T = 2\pi \sqrt{\frac{l}{g}} \]
PURPOSE

To observe SHM (Simple Harmonic Motion) with regard to position, velocity, acceleration, mechanical energy, and phase plots.

THEORY

SHM is a powerful concept found in many ways throughout nature. At its most basic level, SHM requires a restoring force which causes a system to oscillate about an equilibrium point.

We will use a vertical mass-spring system. The mass, when suspended from the spring, has an equilibrium position. If we displace the mass beyond the equilibrium position, the restoring force of the spring will bring it back to equilibrium; but the system now has kinetic energy at the equilibrium point; so it continues past that point and the spring once again pulls (or pushes) the mass back to equilibrium. We end up with a classic case of an oscillation which follows a sine curve.

For the mass-spring system the restoring force is provided by Hooke’s Law. $F = -kx$. (Using a simplified system where the spring/mass system is horizontal, we can set Hooke’s Law equal to Newton’s Second Law of Motion ($F = ma$) giving us $ma + kx = 0$.

$$
ma + kx = 0 \\
\ddot{a} + \frac{k}{m}x = 0 \\
\ddot{x} + \frac{k}{m}x = 0
$$

Where $\ddot{a} = \ddot{x} = \frac{d^2x}{dt^2}$

A subject called Differential Equations addresses the solution of this equation. Basically you have some function $x(t)$ that when multiplied by $\frac{k}{m}$ and added to the second derivative of $x$ you get zero. If we set $x(t) = A \cdot \cos(\sqrt{\frac{k}{m}} \cdot t)$, this function of $x$ satisfies the differential equation.
From this we can conclude that \( x(t) \) will look like some cosine (or sine) function whose angular frequency is \( \sqrt{\frac{k}{m}} \). The linear frequency (Hz = \( \text{cycles} \text{ s}^{-1} \)) would then be \( \frac{1}{2\pi} \sqrt{\frac{k}{m}} \). And the Period \( (T = \frac{1}{f}) \) becomes

\[
T = 2\pi \sqrt{\frac{m}{k}} \quad (1)
\]

This is the Period (T) (seconds cycle) for the mass-spring system. If the mass gets bigger, the period increases. If the spring constant (strength of spring) is larger, the Period is smaller.

What about Energy? The Mechanical Energy of the mass-spring system should be constant. Potential Energy should give rise to Kinetic Energy and visa versa. The total mechanical energy of the system should be constant.

Kinetic Energy is simply

\[
KE = \frac{1}{2}mv^2 \quad (2)
\]

Potential Energy is a little trickier. For a vertical spring, like we will use, we must deal with the potential energy due to gravity plus the potential energy of the spring.

\[
U = \frac{1}{2}ky^2 + mgh + U(0)
\]

Where \( h \) is relative to a point we can pick. And \( y \) is the position of the spring relative to its position no-mass.

But, if we choose \( h \) properly, and make an observation that \( mg = -ky_{\text{equilibrium}} \) when the mass is at rest and hanging from the spring, and perform a substitution for \( y' = 0 \) at the equilibrium position such that \( U(0) \) is at that equilibrium point, then we can “hide” the gravitational potential within the spring potential equation. Our new potential equations becomes simplified to the following equation.

\[
U = \frac{1}{2}ky'^2
\]

But, to make the potential energy equation compatible with our motion sensor (positioned below the mass) we can make this substitution \( y' = x - x_o \); where \( x \) is the position as measured by the motion sensor; and \( x_o \) is the position for when the mass-spring system is at rest.

\[
E_{\text{total}} = \frac{1}{2}mv^2 + \frac{1}{2}k(x - x_o)^2 \quad (3)
\]
EQUIPMENT NEEDED

- Computer, SW750 Interface, Capstone
- Motion Sensor (CI-6742)
- Mass Hanger (Paperclip with large washer)
- table Clamp, 2 Rods, Right-Angle Clamp
- Meter Stick and 2x4 surface for calibration.
- 1 small spring (about $3 \, \text{N/m}$)

1 PROCEDURE

1.1 Prepare Interface and Software

Turn on the interface. Go to ClassSupport >Lab1 >SHM and double-click on the document titled “SHM.cap”.

1.2 Calibrate Motion Sensor

Click on the Capstone tab titled “Calibration”. Turn the dial on the motion sensor so the face is pointed horizontal. Point the motion sensor toward the block of wood and place it exactly 40[cm] away using the meter stick to verify (see Figure 1.)

Remove the meter stick and click the “Record” button. Let the motion sensor run for about 10-15 seconds. We are left with two values,

- meter stick value = 40 cm (Value measured with the meter stick)
- Capstone value = \textit{will vary} cm (Value measured with motion sensor)

Capstone uses a default speed of sound of 344 m/s, and since the speed of sound varies based on several factors, we will have to use the equation below to figure out what the speed of sound is currently in this room.

- Actual Speed of Sound = $344 \left( \frac{\text{MeterStickValue}}{\text{CapstoneValue}} \right)$
Now that you have the Actual Speed of Sound, click on the Hardware Setup button to the far left of the graph. An image of the black Interface box with a motion sensor below it will appear. Click on the “Motion Sensor Image” then click on the “Properties” button. A new dialog box will appear, there will be a value for speed of sound 344 m/s, replace it with your Actual Speed of Sound. Clicking on the Hardware Setup button will collapse the side bar.

1.3 Mass Hanger/Washer
Find the mass of the hanger and washer together (not the spring). Record the mass in the Capstone tab titled “Constants”.

1.4 Position Motion Sensor and Spring System
Hang mass system (hanger and washer) from spring. Place motion sensor under hanging mass. Rotate Motion Sensor using the side pivot knob/dial. Be sure the mass is parallel to table top and not slanted. A slanted mass will cause motion sensor to not properly measure distance.

1.5 Record $x_o$
For this step you should be on the Capstone tab titled “Determine $x_o$.” You will record one run of data and analyze that run in many different ways. Be certain the mass is motionless. Gently lift the mass from underneath by about 3 cm. Let go and allow mass/spring system to stabilize into a nice, smooth motion. If motion is not simply up/down but also side to side or amplitude is large enough to give noisy or messy data, then try again. Once the spring has oscillated a couple of times, click “Record” and collect data for about 3 minutes. Find the mean value and record it on the constants page as $x_o$.

1.6 Find $T$ and $k$
For the steps you should be on the Capstone tab titled “Determine $T$ and $k$.” You will use data already recorded in previous step. Use the “Data” button within the Graph’s toolbar to show the data.

1.7 Change display scale to see 10 Cycles
Use the mouse cursor to hover over the values of the Time axis, this is where the mouse cursor turns to a double-arrow ($\leftrightarrow$). You can drag the scale left or right. We want to scale our Time axis to only see 10-12 cycles of data.
1.8 Measure the Period
You will use the Multi-Coordinates Tool to find the period.
To add a Multi-Coordinates Tool:
1. click add “a coordinates” button
2. choose “Add Multi-Coordinates Tool selection”
The Tool will give you the x value (Time) where ever you place it on the graph, in this case you will place it at the first peak of your 10 cycles. You can have multiple “Multi-Coordinates Tools” on the graph by repeating steps 1 and 2, we will need two “Multi-Coordinates Tools” to find Total time for 10 cycles. Place the second tool on the last peak of your 10 cycles. Use the Total Time value to find the Period, time for one cycle. Record this as the Period (T) on the constants page.

1.9 Calculate $k$
Use $T = 2\pi \sqrt{\frac{m}{k}}$ to calculate $k$. Record $k$ on the constants page. What are the units of $k$?

1.10 Configure Calculator with Constants
Open the Calculate window by clicking the calculate button in the Capstone tab “Determine $T$ and $k$”. One you have clicked on it you will need to fill in your value for $k$, $x_0$ and $m$.

1.11 Observe Plot of $x$, $v$, and $a$
For this step you should be on the Capstone tab titled “$x$, $v$, and $a$.”. You will need to bring the data in using the data button for each graph ($x$, $v$, and $a$). Don’t forget you can change the x axis display scale by hovering over the axis and producing the double arrow (→). Annotate the following using the “Annotate button”:
- Label the equilibrium position. Just point to it and name it.
- Label the max $|$velocity$|$ for one cycle. Note with which poision value they correspond.
Can you recognize the position curve as a Cosine (or Sine) and the velocity is shifted to the left by $\frac{1}{4}$ wavelength and acceleration is shifted again?
1.12 Observe Energy Plots

For this step you should be on the Capstone tab titled “Energies.” Bring in the data using the “Data” button on the plots for U, KE, and Etotal. Annotate any points that are interesting to you. (Please don’t consider noisy data points as interesting.)

1.13 Phase Plots

For this step you should be on the Capstone tab titled “Phase Plots.” Bring in the data to these plots. What do you notice in the Phase Plots generated? Why does each shape look like it does? Why does Potential Energy look like it does? {It is so thin compared to the others that are filled in completely.} What can you say about conservation of energy for the mass-spring system? Answer these questions in the Discussion page.

1.14 Discussion

For this step you should be on the Capstone tab titled “SHM Discussion.” Write a paragraph or two discussing SHM in light of what you have observed.
SHM

Lab Station Number:

Names Sorted Alphabetically (by last names)

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Date Data Was Taken:

Instructor's Name:

Lab Time:
examples: T8, W4, R10

Pages are stapled in upper left corner and are in this order (5pts):
Instructors are encouraged to grade this on the lab computers during class.

Student, please remember, POSTLAB is due at end of class this same day!

Properly Finish This CoverSheet (5pts)
List The Constants with units and show math, if math was needed.

mass = 
X₀ = 
T = 
k = 
HOW TO READ A VERNIER SCALE

These samples are obviously not to scale. They are scaled to 170%.

The vernier arrow is at 0.000 cm.

The vernier arrow is at 1.1?? cm.

The 4.0 mark on the vernier lines up with the cm scale.
So the vernier arrow is pointing at 1.140 cm.

The vernier arrow is at 2.6?? cm.

The 8.5 mark on the vernier lines up with the cm scale.
So the vernier arrow is pointing at 2.685 cm.

Be certain that "Scale to Fit" is turned off in your print dialog if you want the cm scale to the left to be a real cm scale!
For the curious, this is how I made the vernier scale for the cm(mm) ruler:

Notice the vernier scale has 20 marks that match up in the end with 39 marks on the major scale.  
20 vernier divisions * ?% = 39 major divisions  
39/20 = 1.95% will create the length of the vernier scale.  
So, scale the length of 20 major divisions by 1.95%. That is the vernier scale.  
20 divisions means that when I move the vernier scale so that the first vernier mark matches the 1st major division,  
the movement is worth 1/20th of a major division.  
1/20 = 0.05 of a major division.  
Each vernier is worth 1/20 * 1 millimeter = 0.05 mm.  

A vernier scale acts like a magnifying glass against a larger scale. It allows you to find out the “inbetweeness” when the object being measured doesn’t match up exactly with a mark on the main scale.

This vernier scale claims to measure within one half of one hundereth of a cm. I realize this is rather silly; for one should purchase a more precise scale if 50 µm measurements are needed. But, hey, this one is free. You get for which you pay.